

Binary Search

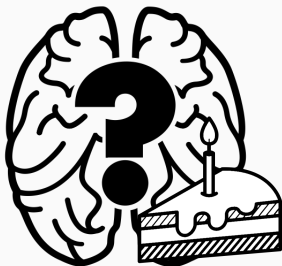
Joël Mathys

5. November 2016

Swiss Olympiad in Informatics

Introduction to Binary Search

Guessing Game



My date of birth

Guessing Game Reloaded



Numbers

1, 2, 3, ..., 10'000

Only 15 Questions

Guessing Game Strategies

- Go through every Element
 - $O(n)$



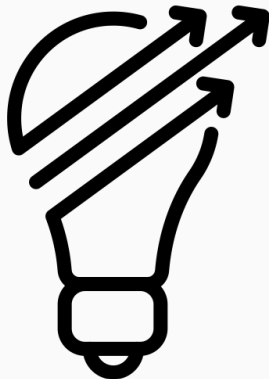
Guessing Game Strategies

- Go through every Element
 - $O(n)$
- Divide into Categories
 - $O(\sqrt{n})$



Guessing Game Strategies

- Go through every Element
 - $O(n)$
- Divide into Categories
 - $O(\sqrt{n})$
- Divide into two equal Halves
 - Binary Search
 - Much faster



Binary Search Theory

- Elements must have some kind of Order
 - Booleans, consecutive
 - 1110000



Binary Search Theory

- Elements must have some kind of Order
 - Booleans, consecutive
 - 1110000
 - Numbers, asc./desc.
 - 1,4,5,23,123
 - 5,2,-4,-23



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 - 5,2,-4,-23
 - Curves
 - left or right



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- End must be clear



Binary Search Theory

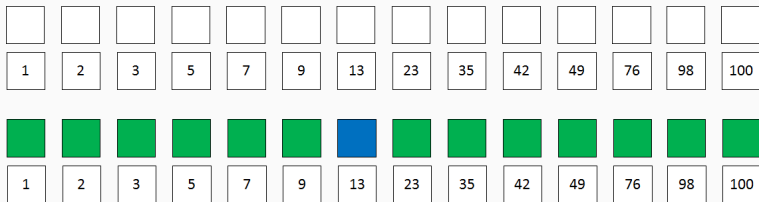
- Elements must have some kind of Order
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 - Numbers, asc./desc.
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 - Curves
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- Next step must be clear
- End must be clear
- Runtime of $O(\log(n)) \approx 10$
- almost constant time



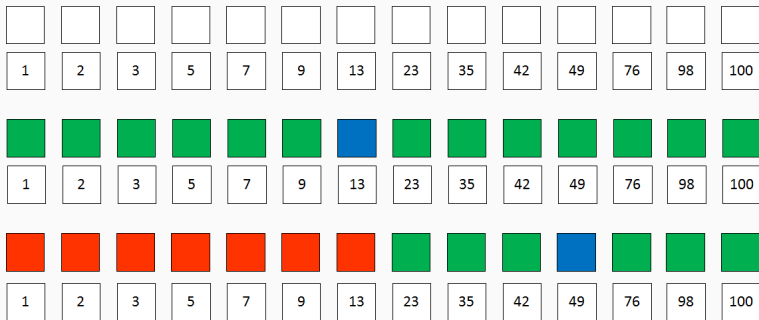
Visualisation

1	2	3	5	7	9	13	23	35	42	49	76	98	100

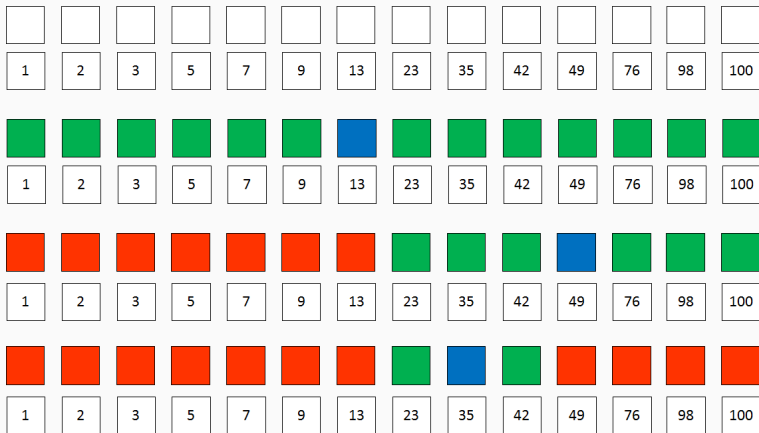
Visualisation



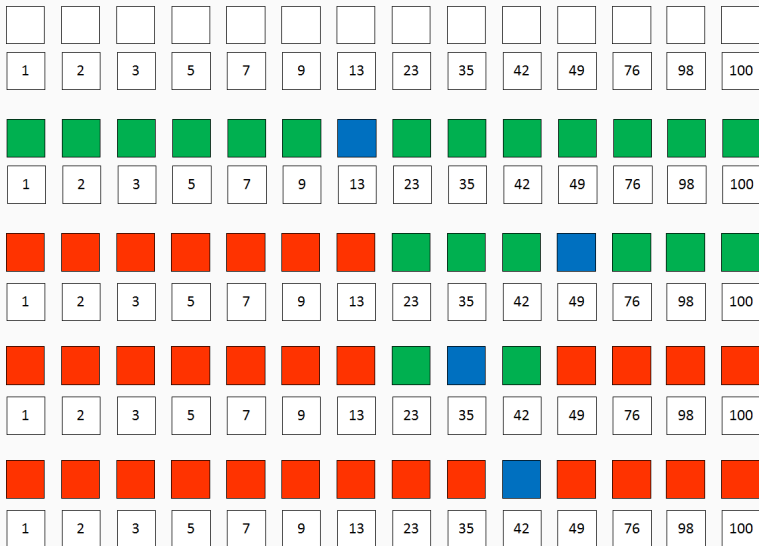
Visualisation



Visualisation



Visualisation



Implementation

```
int bsearch(vector<int> &A, int e){
    int down = 0;
    int up = (int)A.size();
    int middle;
    // down is always <= e
    while(up - down > 1){
        middle = (up+down)/2;
        if(A[middle] <= e){
            down = middle;
        }else{
            up = middle;
        }
    }
    if(A[down] == e){
        return down;
    }
    return -1;
}
```

```
def bsearch(A, e):
    down = 0
    up = len(A)
    middle = 0
    while up-down > 1:
        middle = int((down+up)/2)
        if A[middle] <= e:
            down = middle
        else:
            up = middle

    if A[down] == e:
        return down
    return -1
```

Implementation

Danger

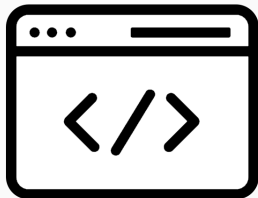
- $\text{down} \neq \text{up}$
- off by one
- false rounding

Good Practise

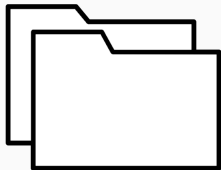
- size of intervall, up-down
- make sure either left or right are always true
- always round the same

```
int bsearch(vector<int> &A, int e){
    int down = 0;
    int up = (int)A.size();
    int middle;
    // down is always <= e
    while(up - down > 1){
        middle = (up+down)/2;
        if(A[middle] <= e){
            down = middle;
        }else{
            up = middle;
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    }
    if(A[down] == e){
        return down;
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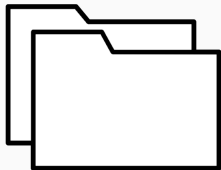
- If you only search Element or Bounds
 - Built in functions of STL Library
 - SOI-Cheatsheet
- Why implement Binary Search ?
 - normally very task dependant
 - not that hard



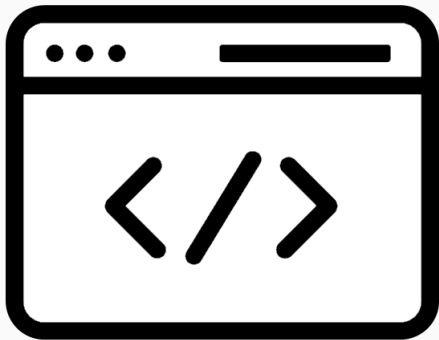
- Treasure
 - Search treasure on a map
 - get directions
 - 1 Dimension



- Treasure
 - Search treasure on a map
 - get directions
 - 1 Dimension
- Loan
 - split tasks upon co-workers
 - which task sizes make sense?
 - how would you distribute the work?
 - how would the first worker begin?







Valar Codelis