Swiss Olympiad in Informatics 2018

Graph Theory and DFS

Dennis Komm

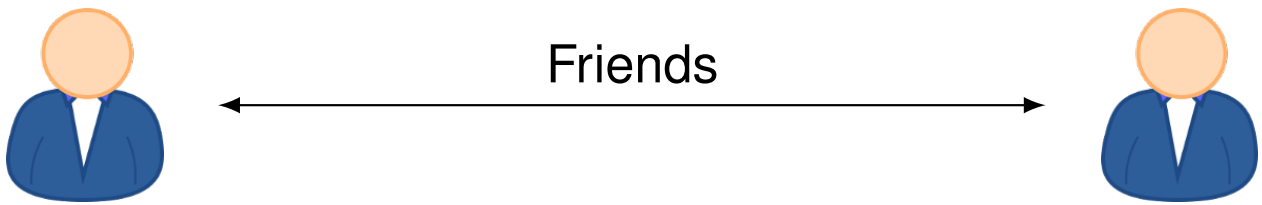
February 12, 2018 – Ftan

Searching in Networks

Social Network

Social Network

Social Network



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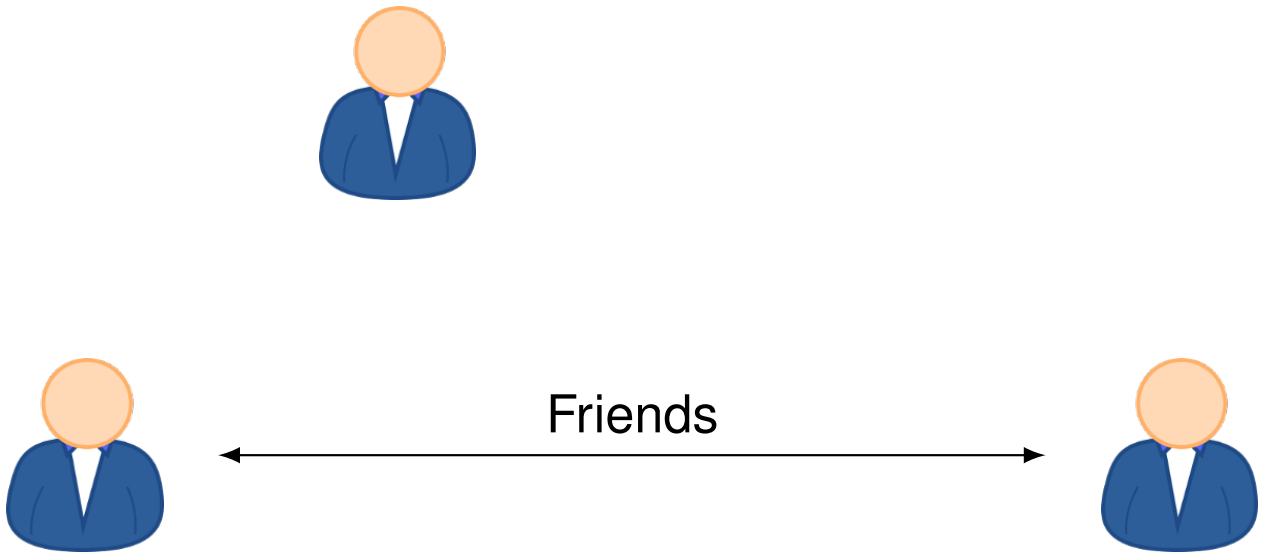

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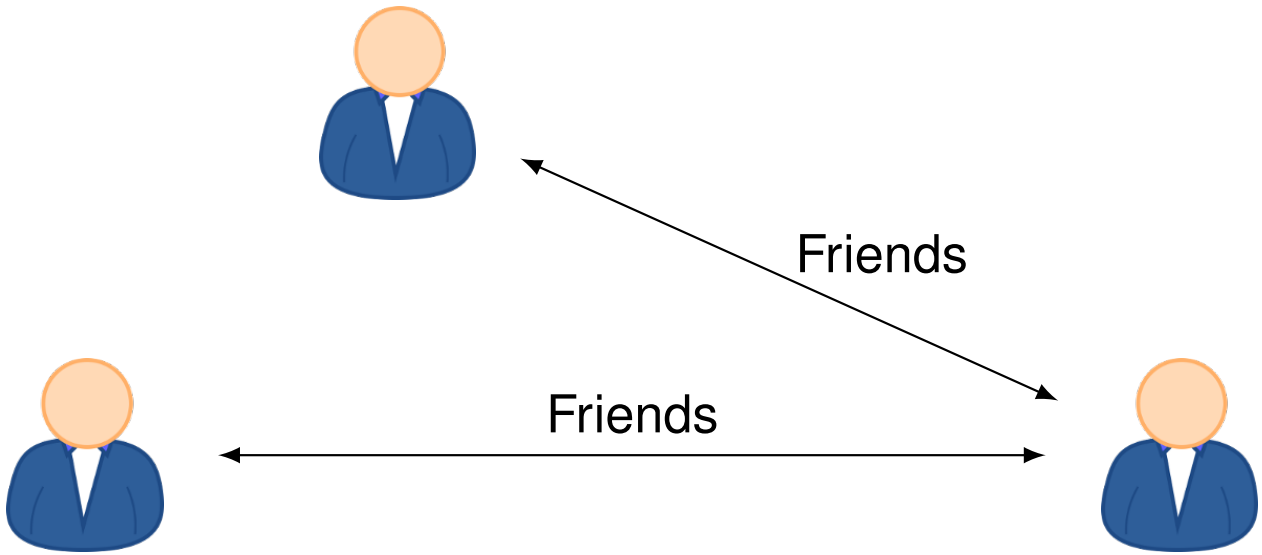

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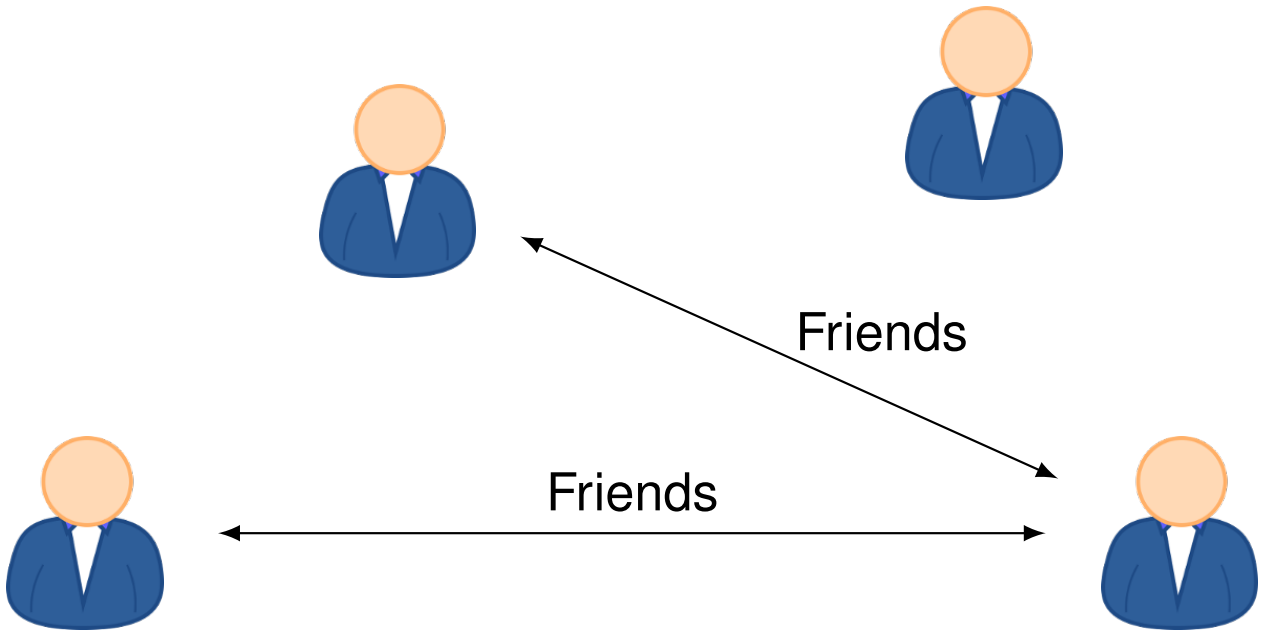

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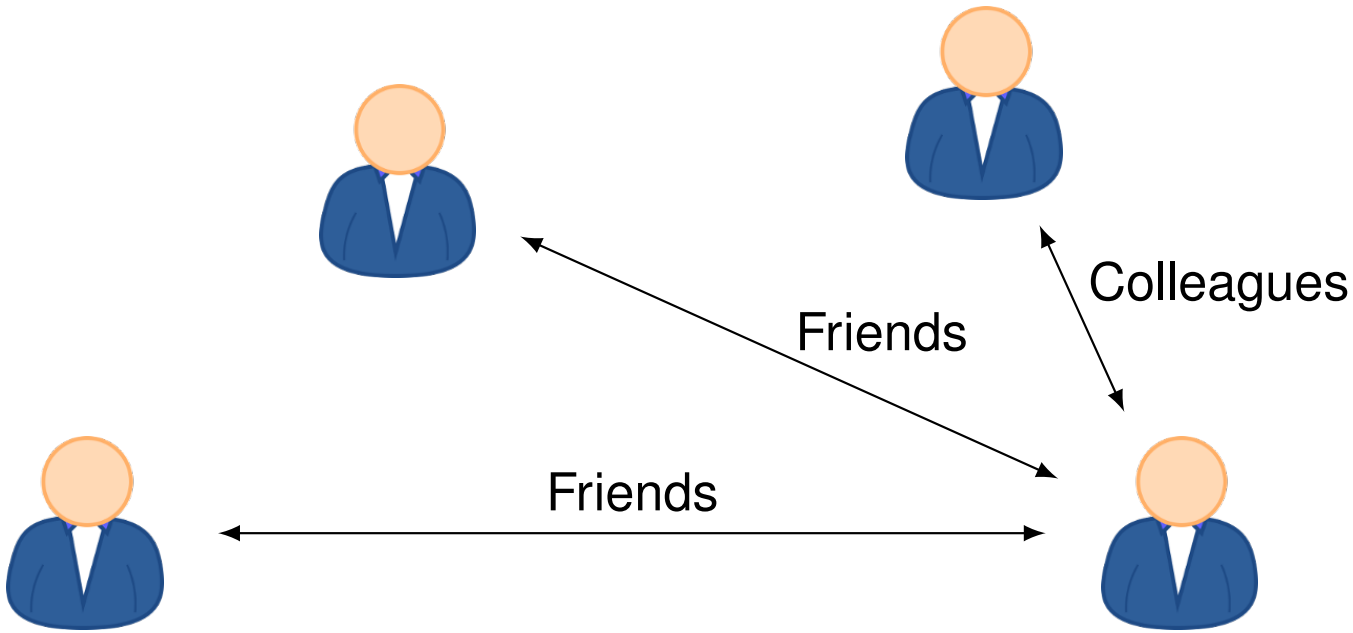

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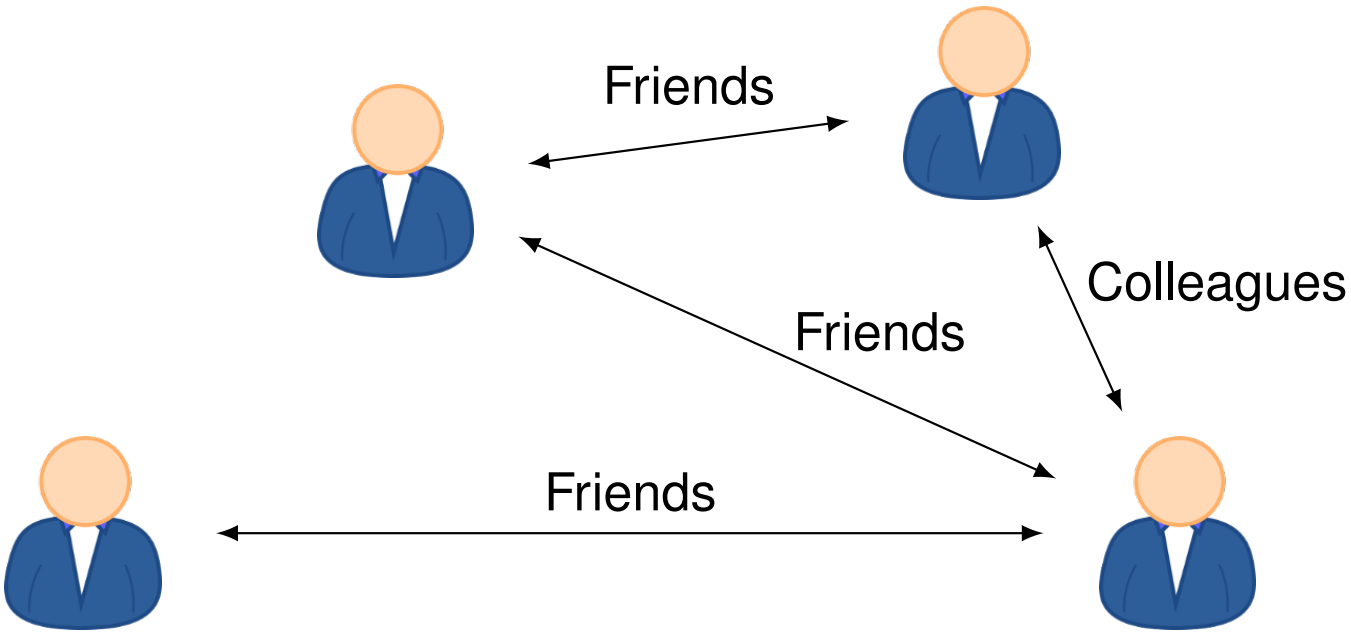

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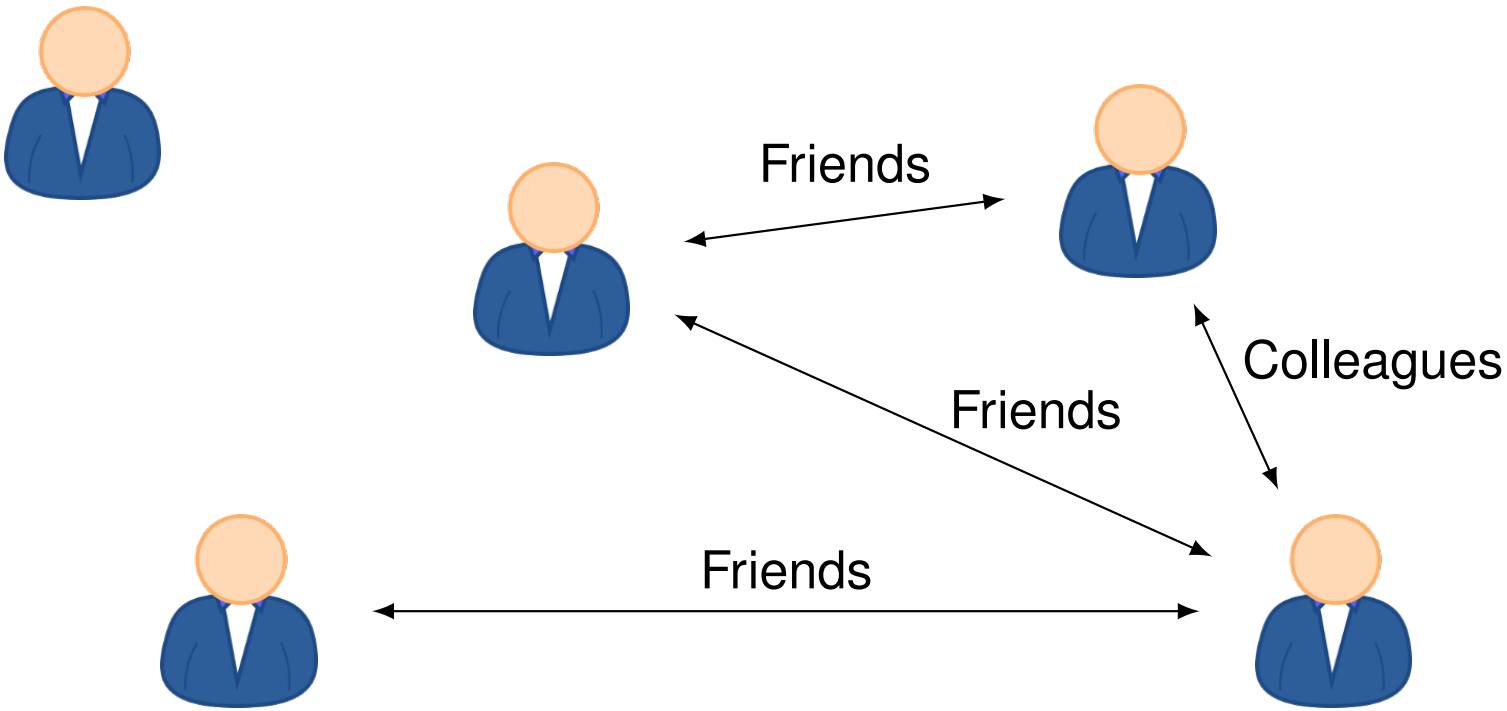

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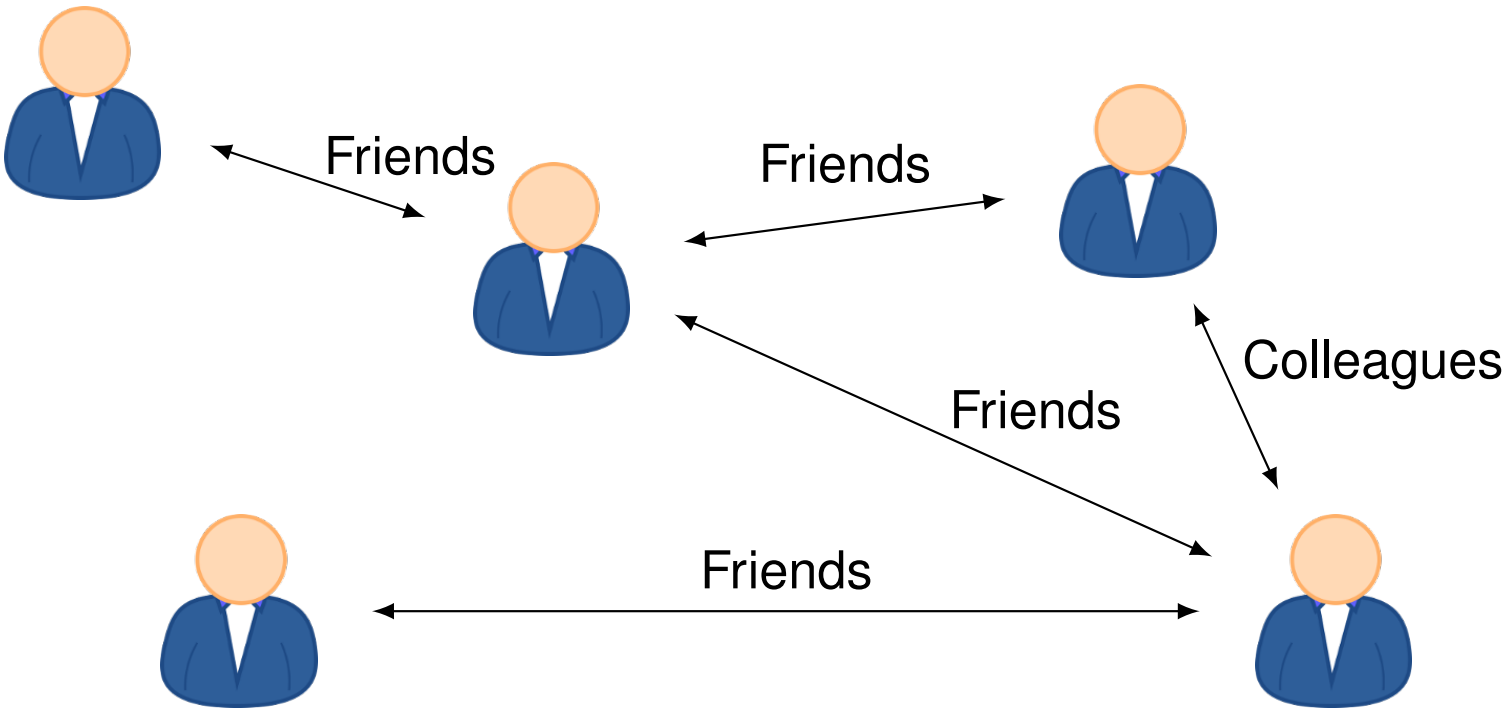

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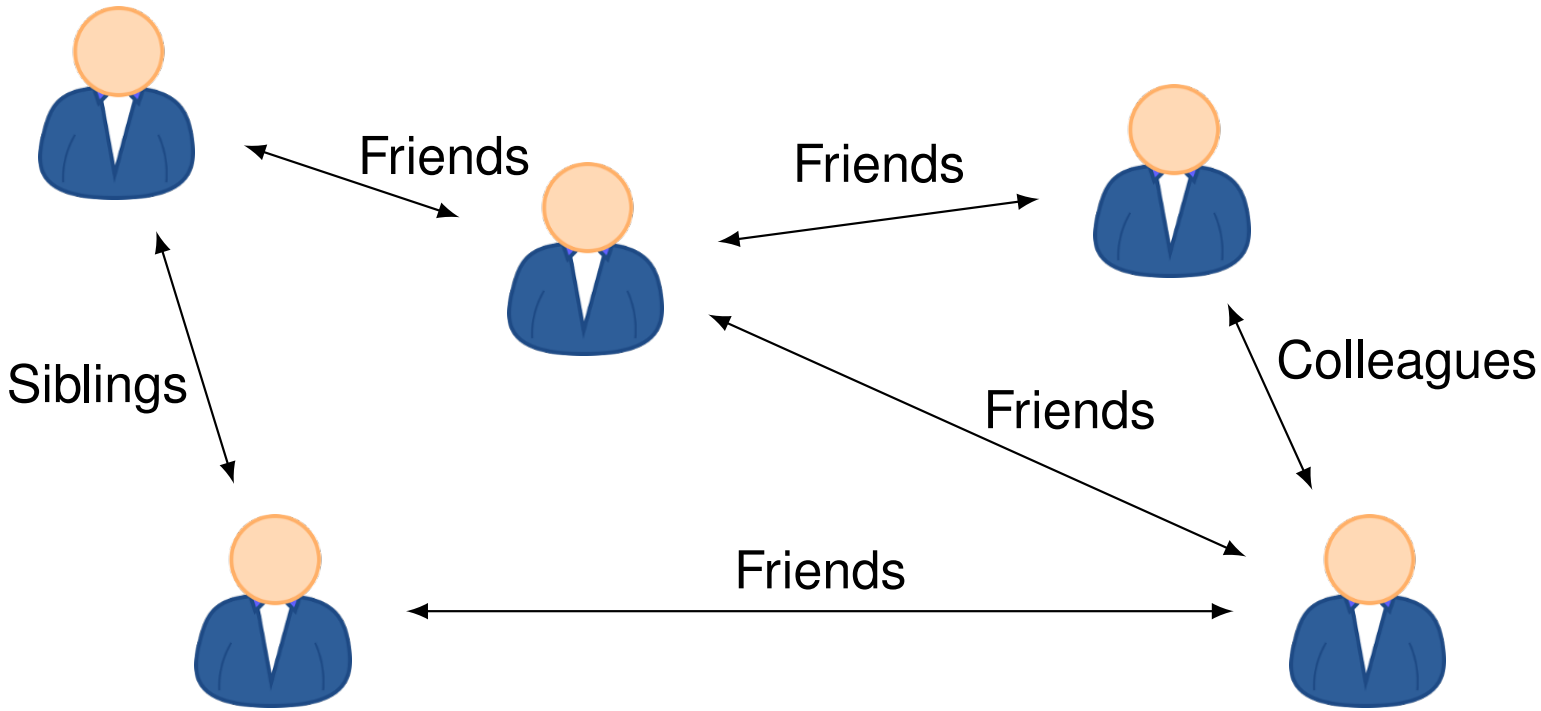

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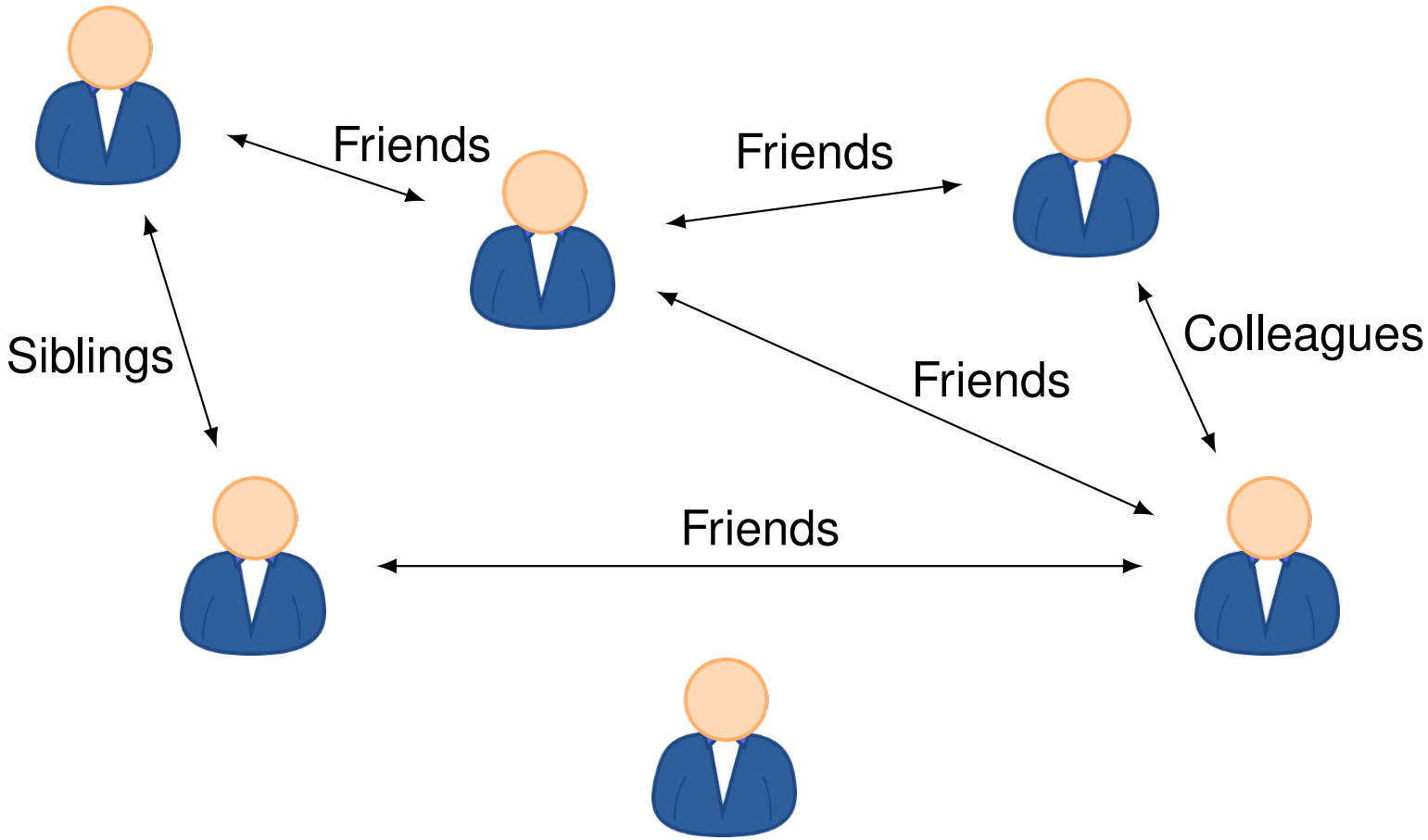

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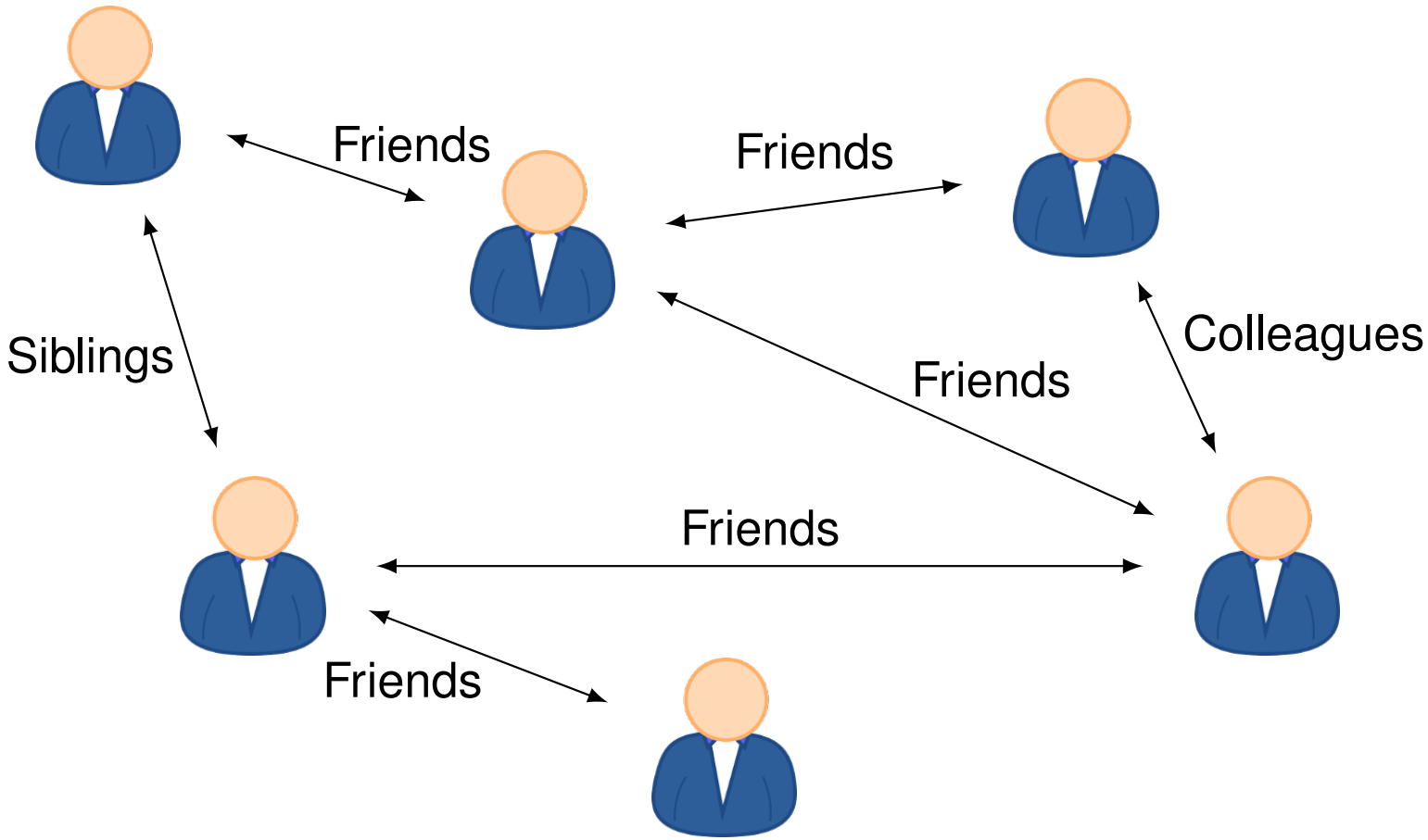

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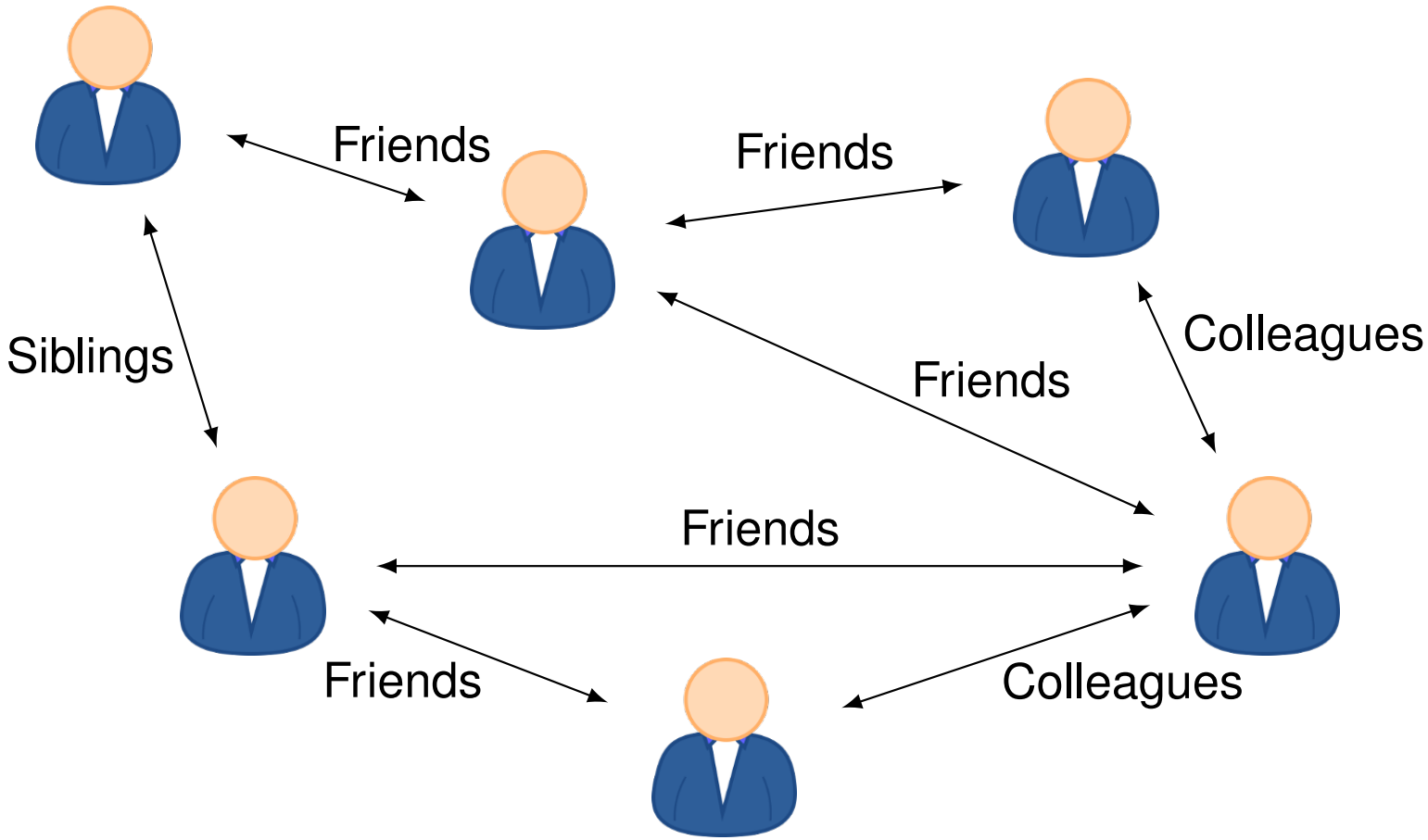

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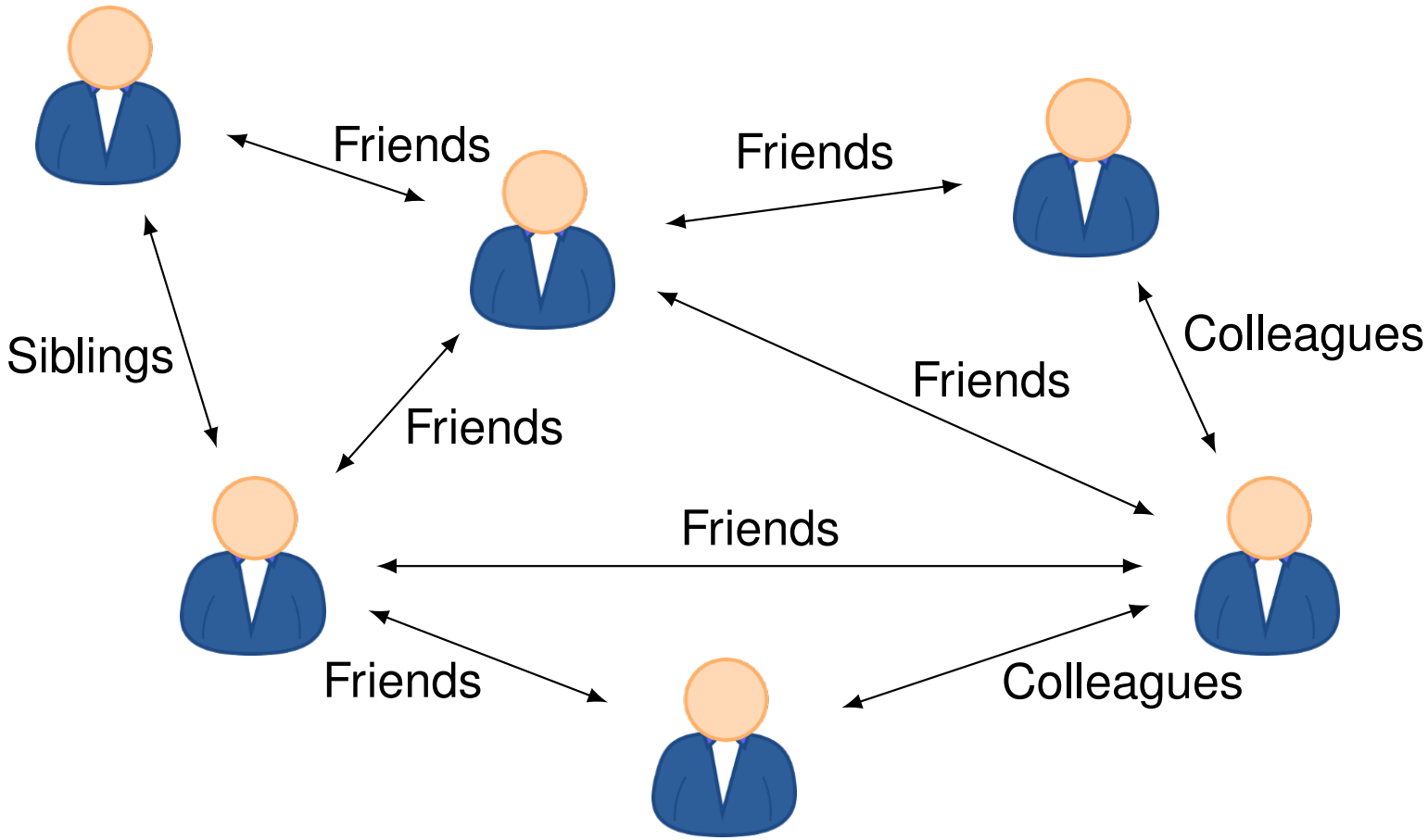

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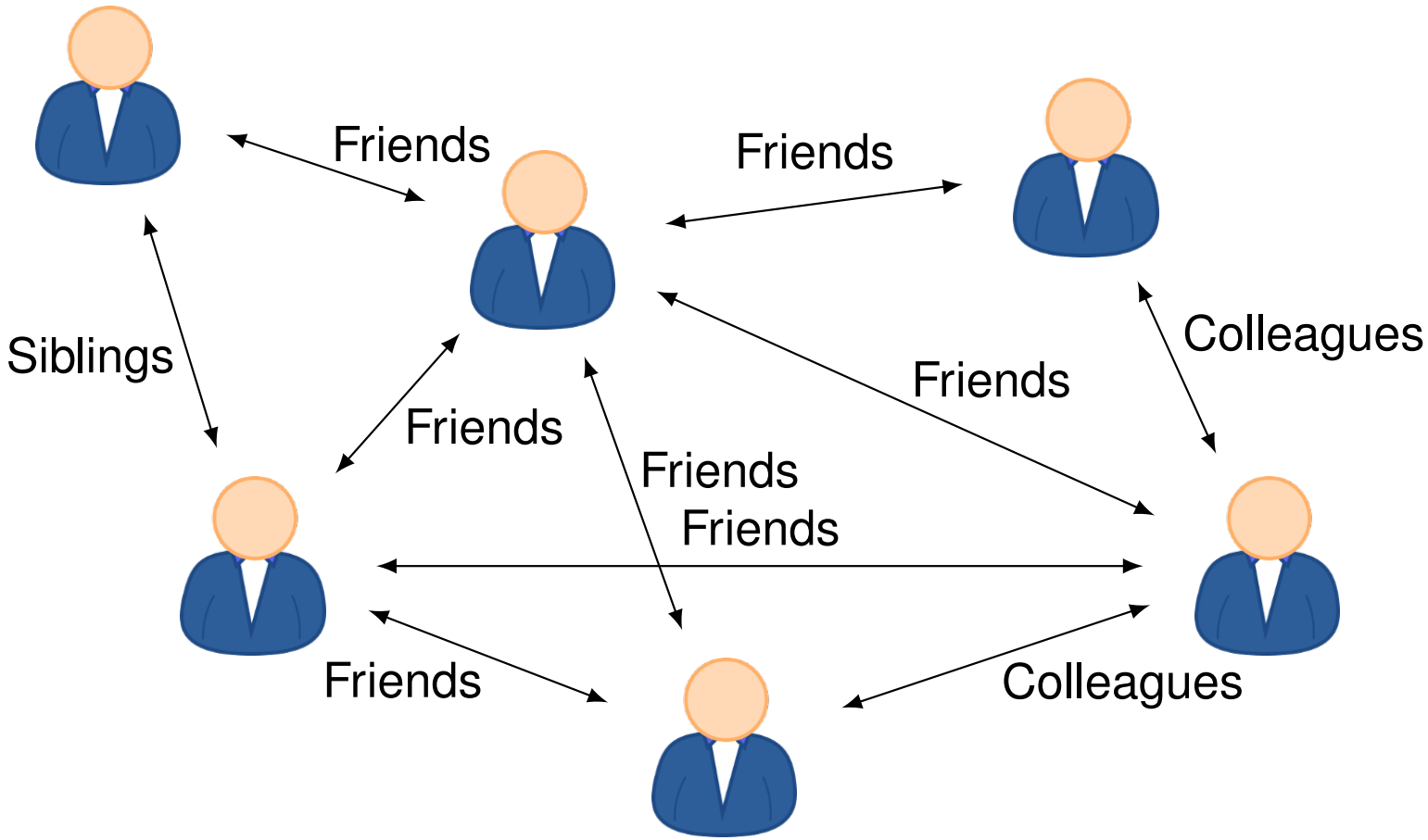
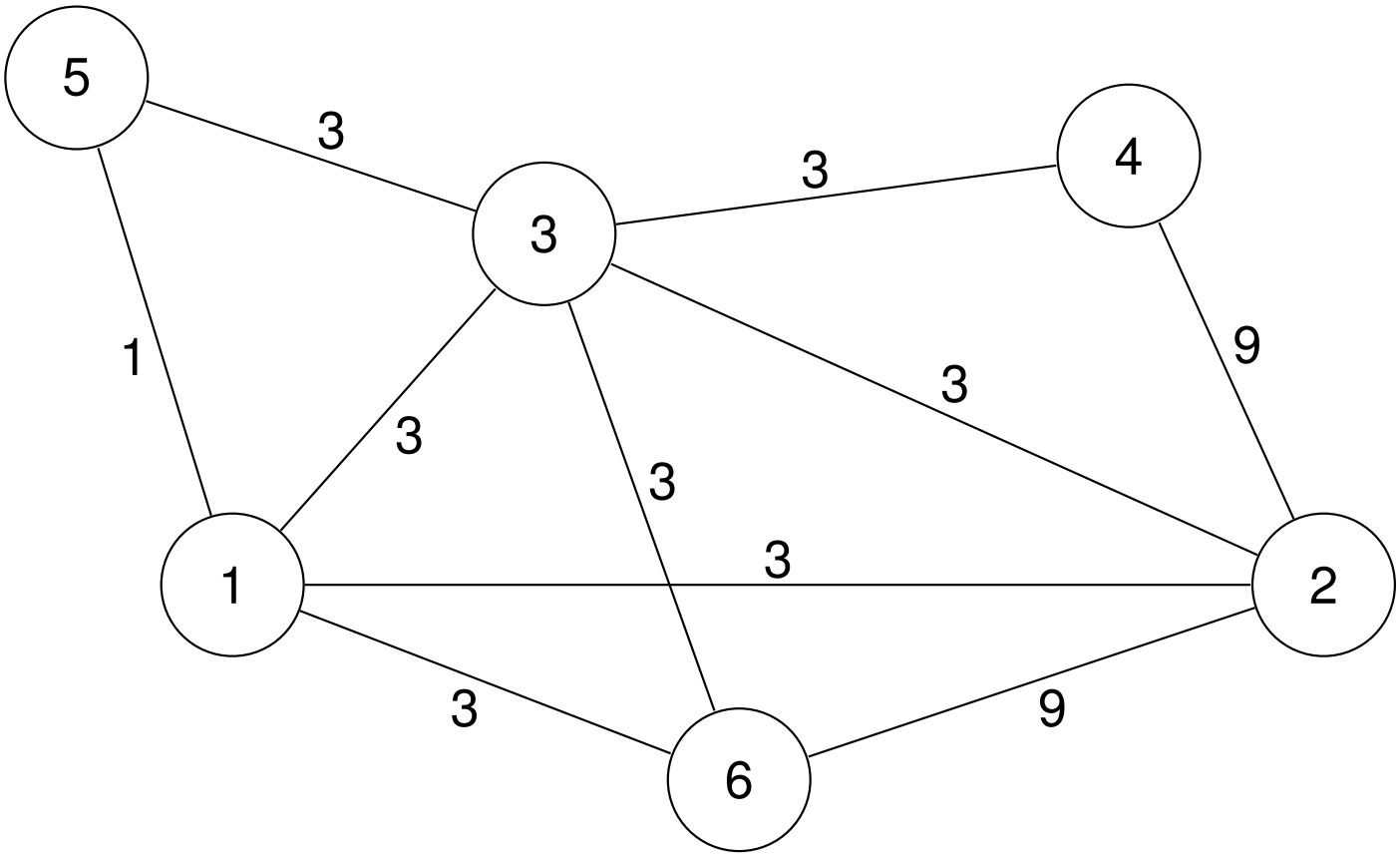


Abstract Modelling



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- Graphs are either **undirected** or **directed**
- Graphs are either **connected** or **unconnected**

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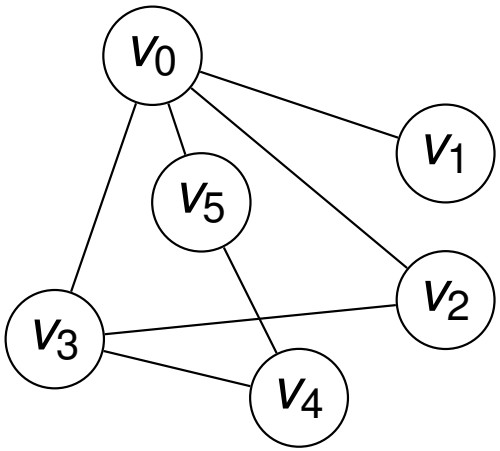
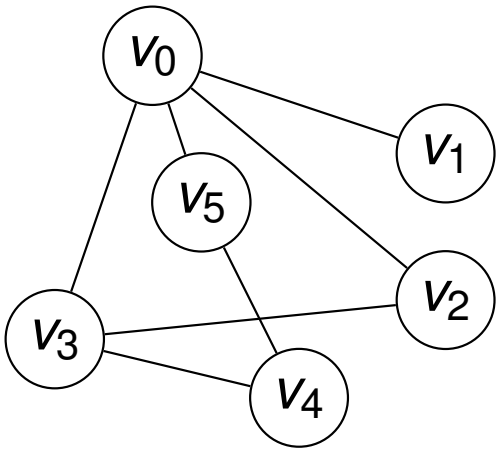


Undirected unweighted graph

Abstract Modelling



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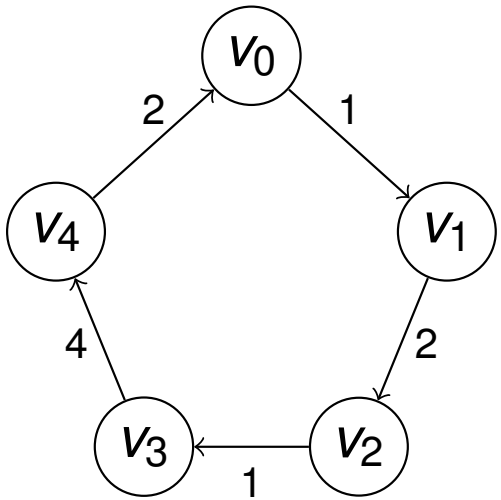

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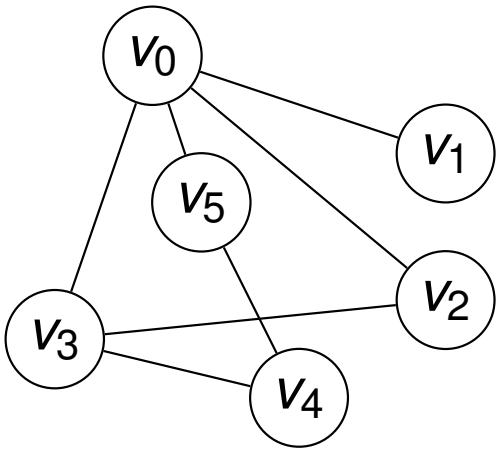

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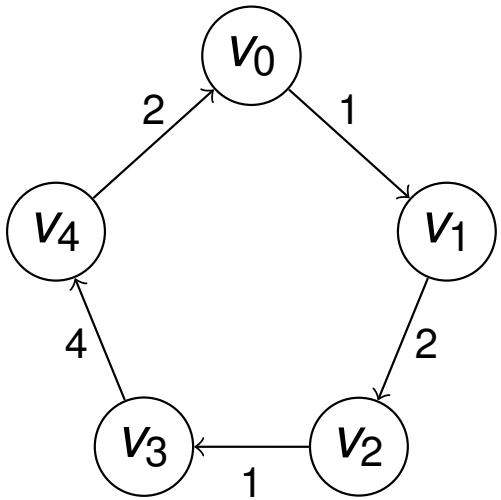
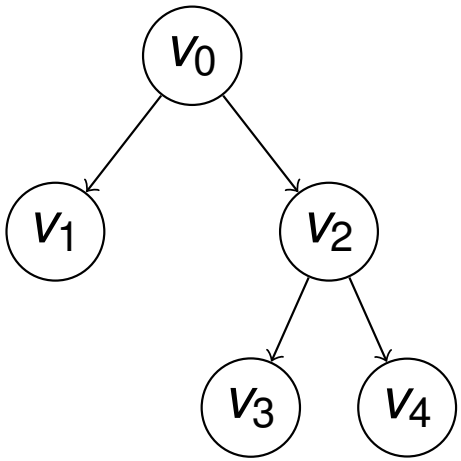
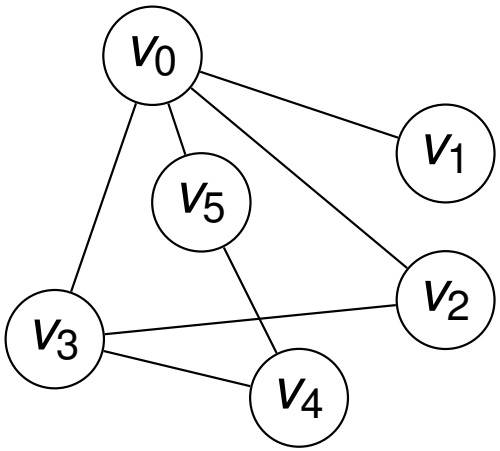


Undirected weighted graph

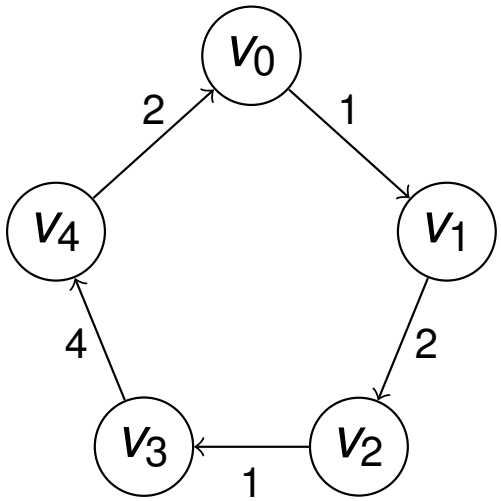


Directed unweighted graph

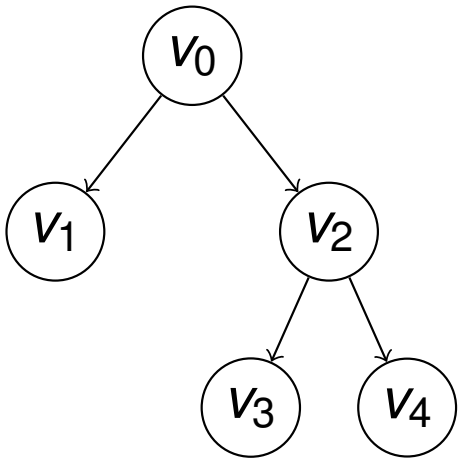
Abstract Modelling



Undirected unweighted graph



Undirected weighted graph



Directed unweighted graph

Which graphs are used depends on what we want to model

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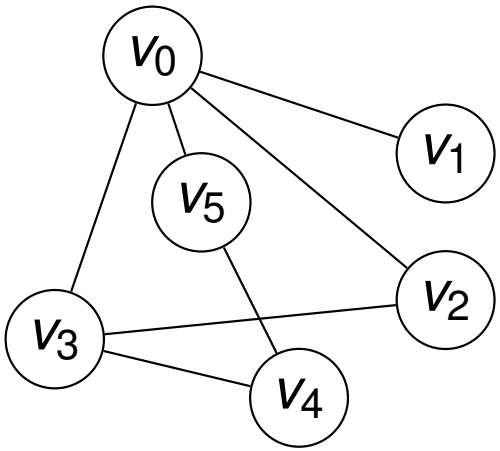

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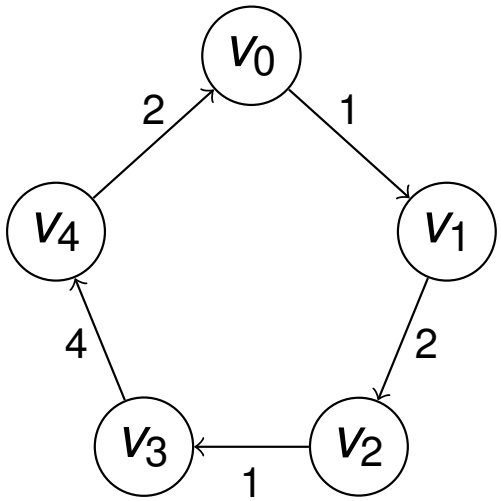

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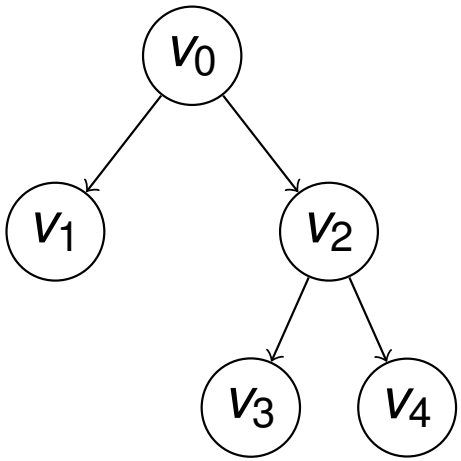


Directed unweighted graph

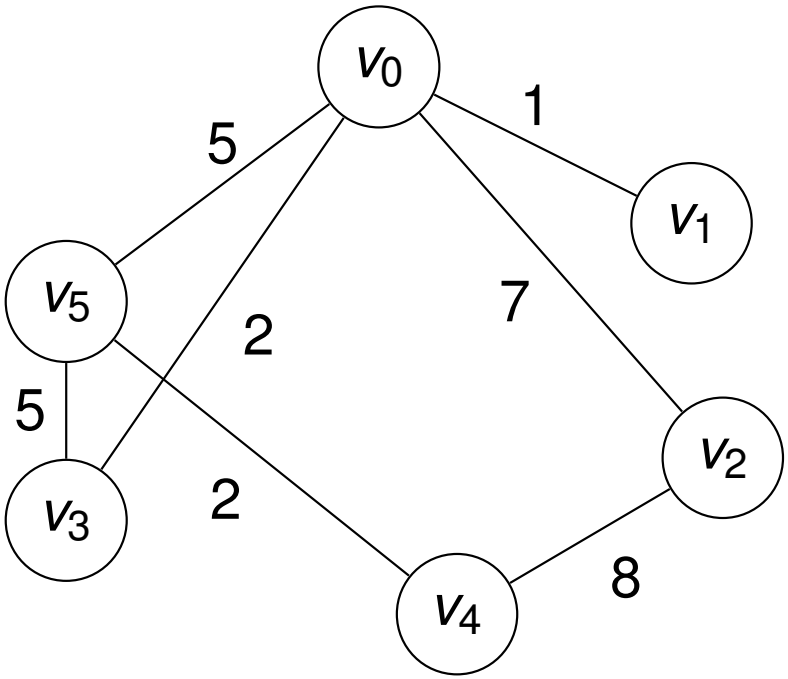
Which graphs are used depends on what we want to model

For simple arguments, we mostly assume connected graphs

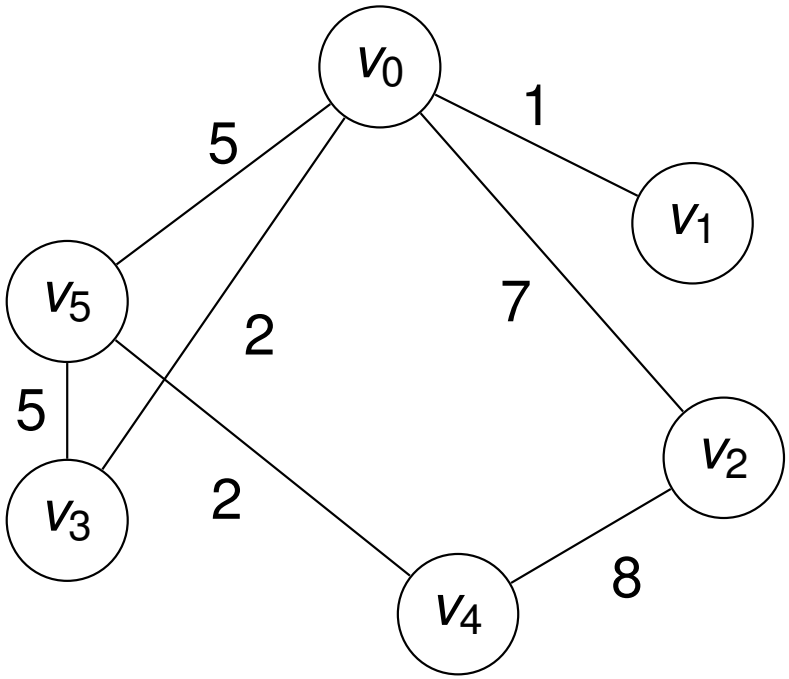
On the Computer

Graphs

Adj. Matrices: Undirected Weighted Graph

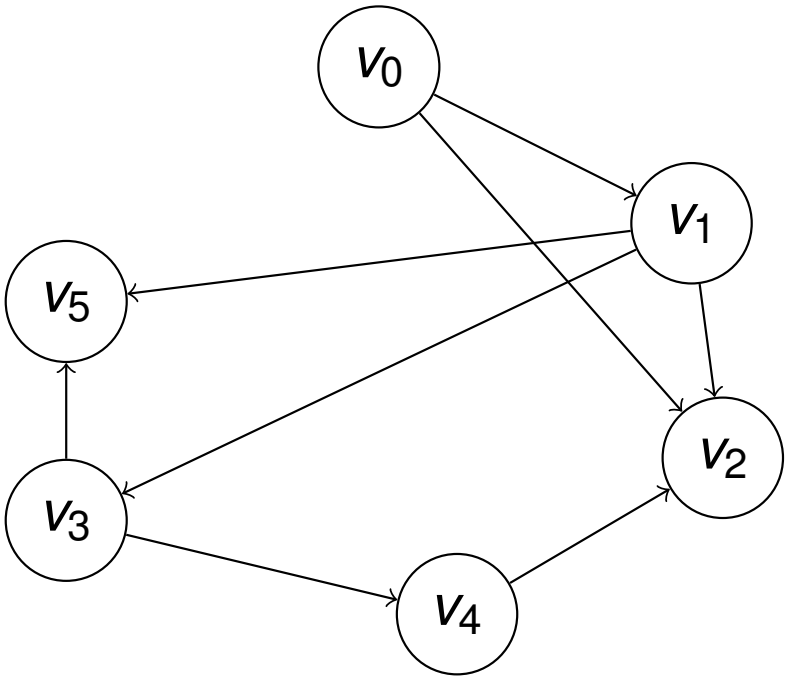


Adj. Matrices: Undirected Weighted Graph



$$\begin{pmatrix} 0 & 1 & 7 & 2 & 0 & 5 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 8 & 0 \\ 2 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 8 & 0 & 0 & 2 \\ 5 & 0 & 0 & 5 & 2 & 0 \end{pmatrix}$$

Adj. Matrices: Directed Unweighted Graph



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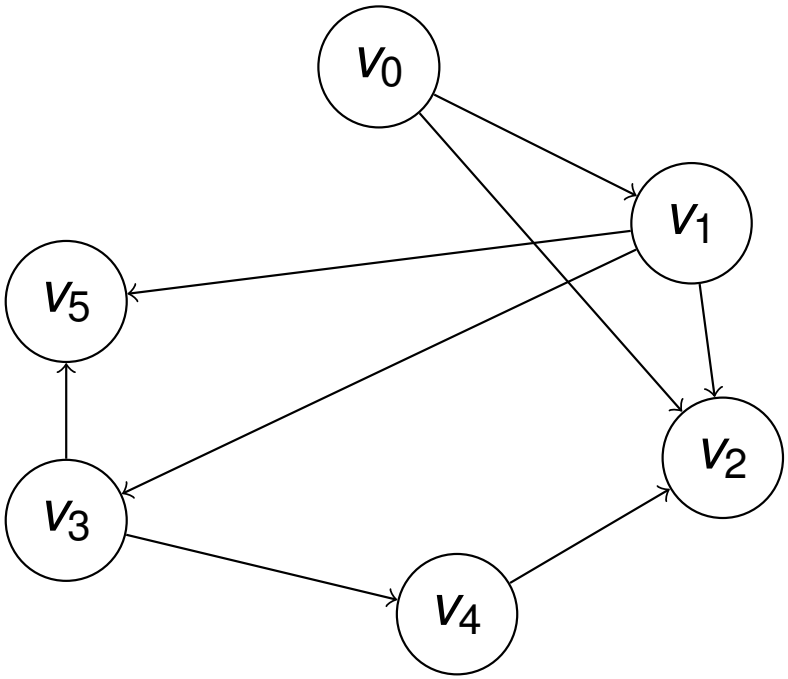
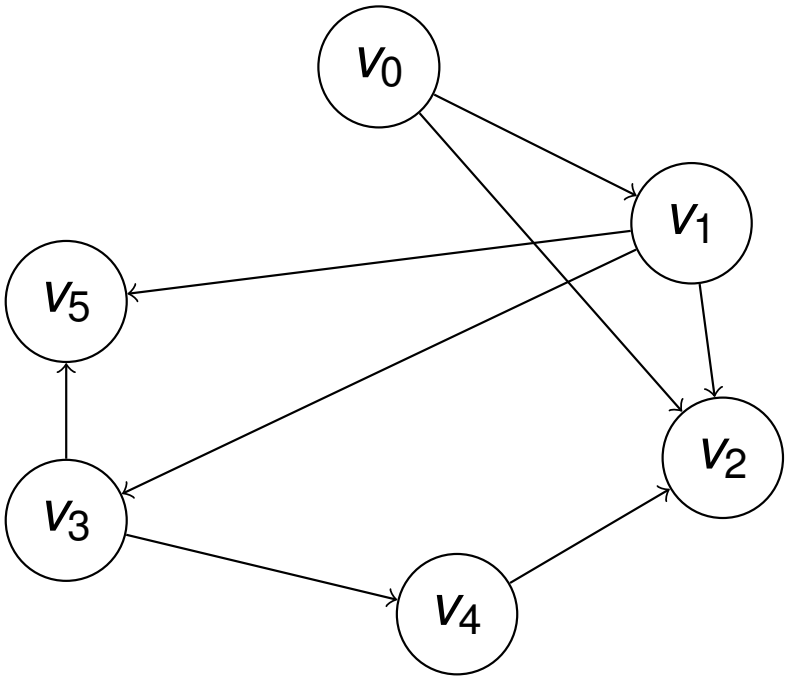


$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Adj. Lists: Directed Unweighted Graph



$((1,2),$
$(2,3,5),$
$(),$
$(4,5),$
$(2),$
$())$

Adj. Matrices and Lists in C++

Use 2-dimensional `vector` of `int`s

Adj. Matrices and Lists in C++

Use 2-dimensional `vector` of `ints`

Matrix: Weighted

```
vector<vector<int>>
G { {0, 1, 7, 2, 0, 5},
    {1, 0, 0, 0, 0, 0},
    {7, 0, 0, 0, 8, 0},
    {2, 0, 0, 0, 0, 5},
    {0, 0, 8, 0, 0, 2},
    {5, 0, 0, 5, 2, 0} };
```

Adj. Matrices and Lists in C++

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    {5, 0, 0, 5, 2, 0} };
```

Matrix: Unweighted

```
vector<vector<int>>
G { {0, 1, 1, 0, 0, 0},
    {1, 0, 1, 1, 0, 1},
    {1, 1, 0, 0, 1, 0},
    {0, 1, 0, 0, 1, 1},
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```

List: Unweighted

```
vector<vector<int>> G
{{1,2},{0,2,3,5},{0,1,4},{1,4,5},{2,3},{1,3}};
```

Adj. Matrices and Lists in C++

Use 2-dimensional `vector` of `ints`

Matrix: Weighted

```
vector<vector<int>>
G { {0, 1, 7, 2, 0, 5},
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```

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vector<vector<int>>
G { {0, 1, 1, 0, 0, 0},
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    {0, 1, 0, 0, 1, 1},
    {0, 0, 1, 1, 0, 0},
    {0, 1, 0, 1, 0, 0} };
```

List: Unweighted (We prefer those)

```
vector<vector<int>> G
{{1,2},{0,2,3,5},{0,1,4},{1,4,5},{2,3},{1,3}};
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Stacks

For iterative DFS

Stacks

- Last-In First-Out

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- `stack<int>` creates stack of integers

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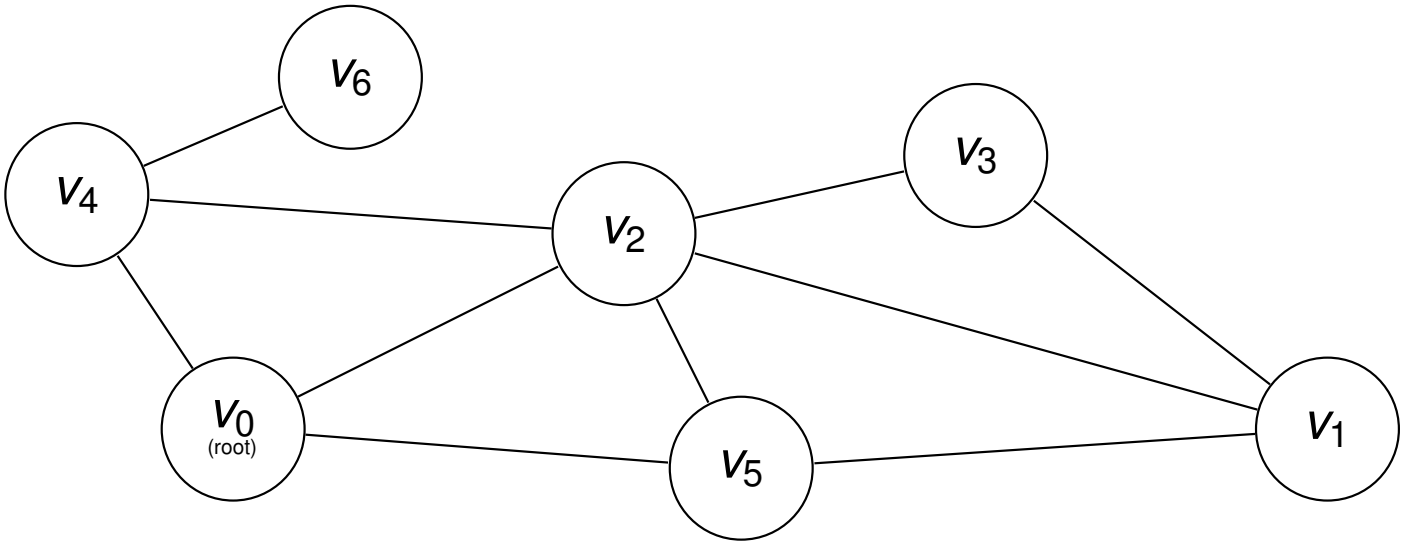
- Last-In First-Out
- `stack<int>` creates stack of integers
- `push(x)` “pushes” `x` into end
- `top()` returns last element
- `pop()` “pops” out last element

```
stack<int> numbers;
numbers.push(0);
numbers.push(4);
numbers.push(7);
cout << numbers.top() << "\n";
numbers.pop();
cout << numbers.top() << "\n";
```

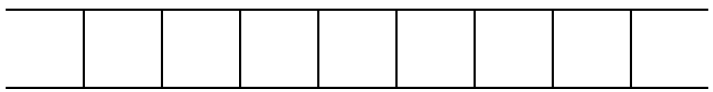
Depth-First Search

in undirected graphs

DFS with a Stack

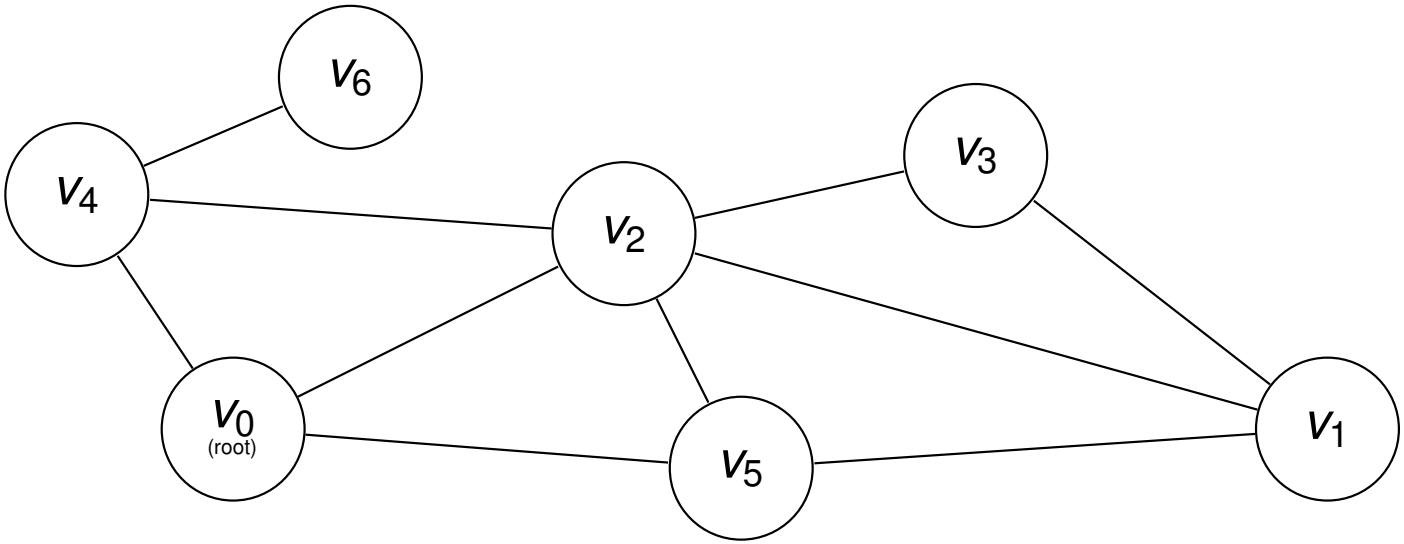


Stack:

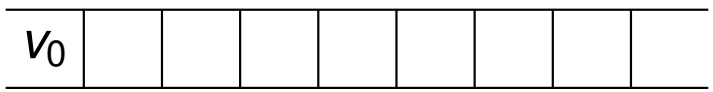


Output:

DFS with a Stack

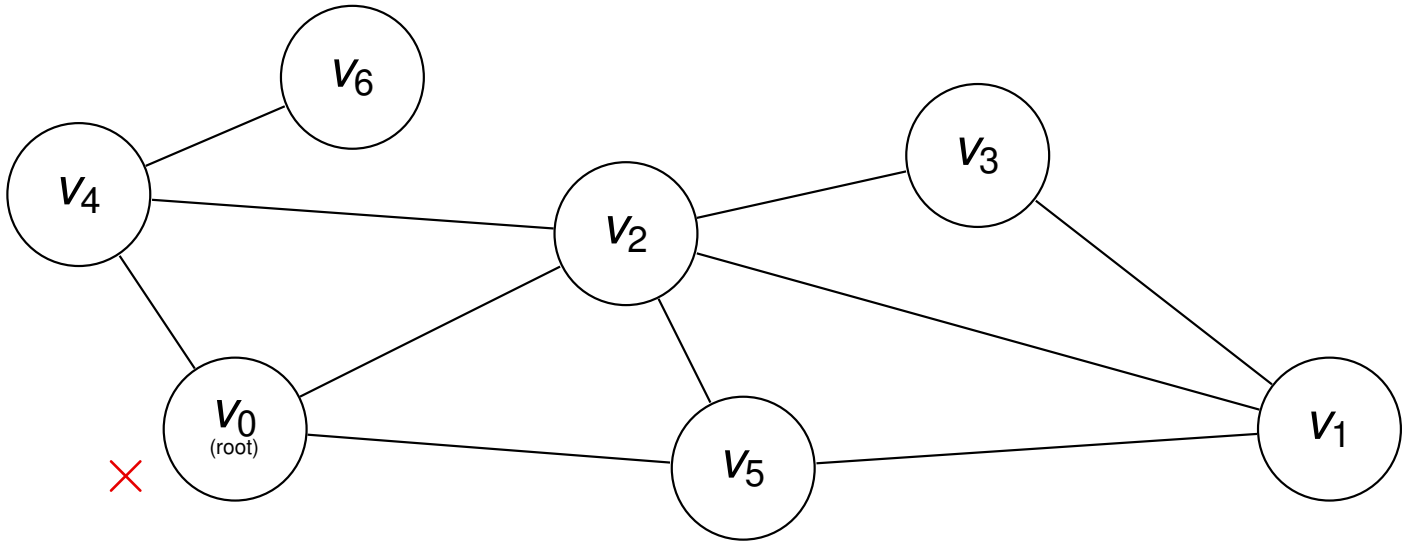


Stack:



Output:

DFS with a Stack



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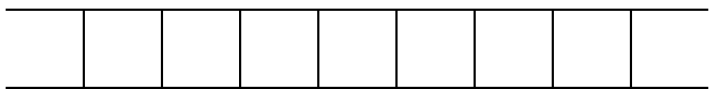
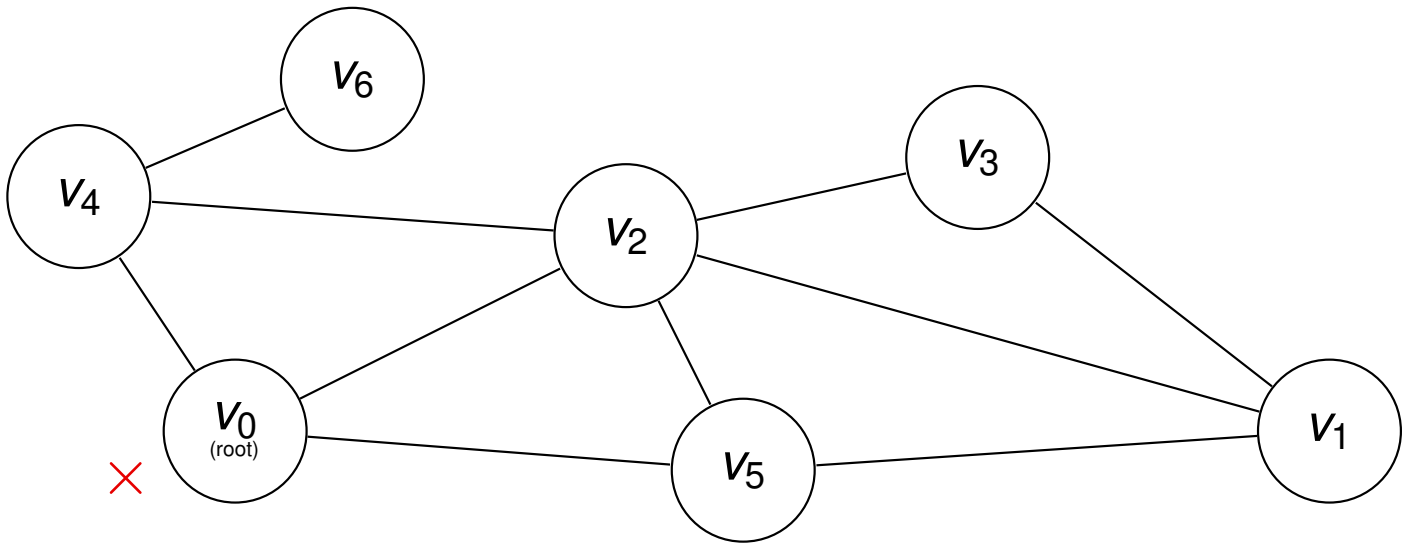


Output: v_0

DFS with a Stack

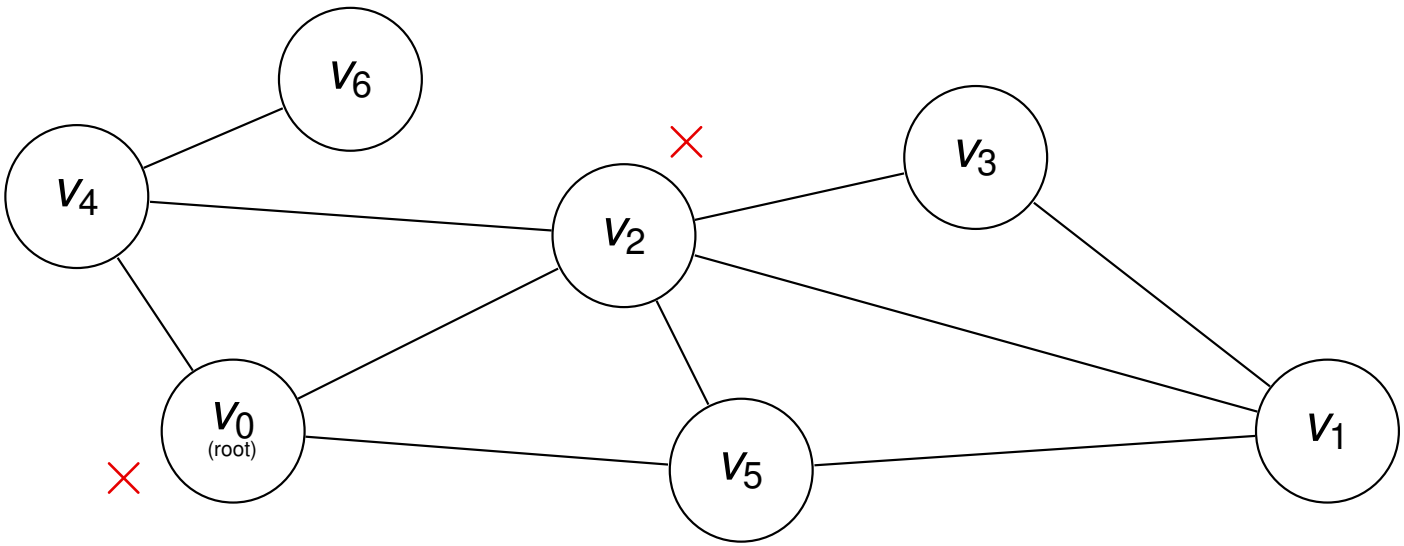


Stack:

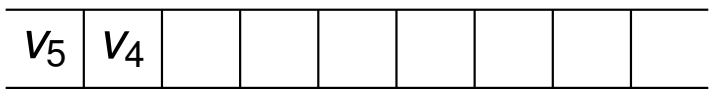
v_5	v_4	v_2						

Output: v_0

DFS with a Stack

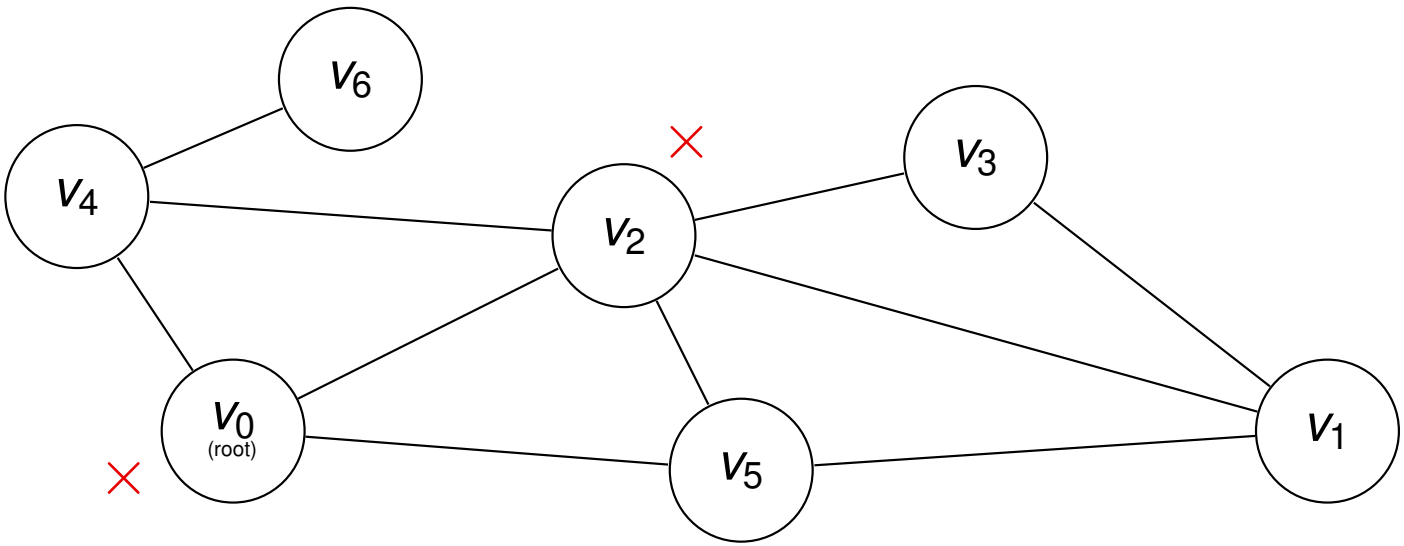


Stack:



Output: v_0 v_2

DFS with a Stack

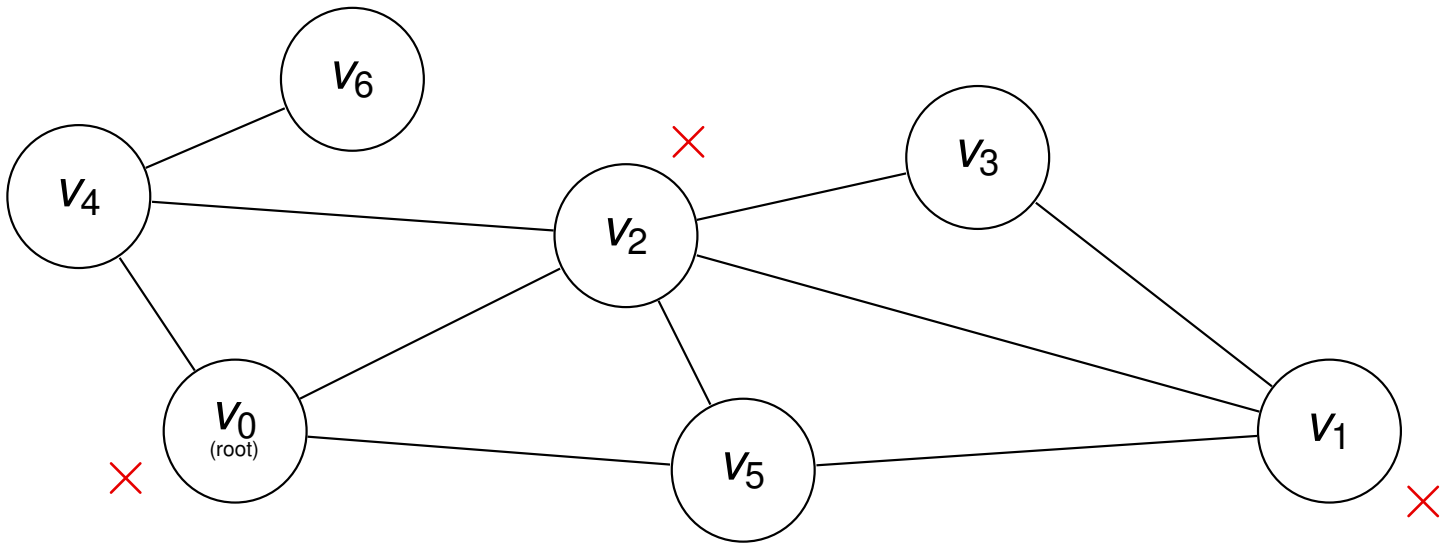


Stack:

v_5	v_4	v_5	v_4	v_3	v_1			

Output: v_0 v_2

DFS with a Stack

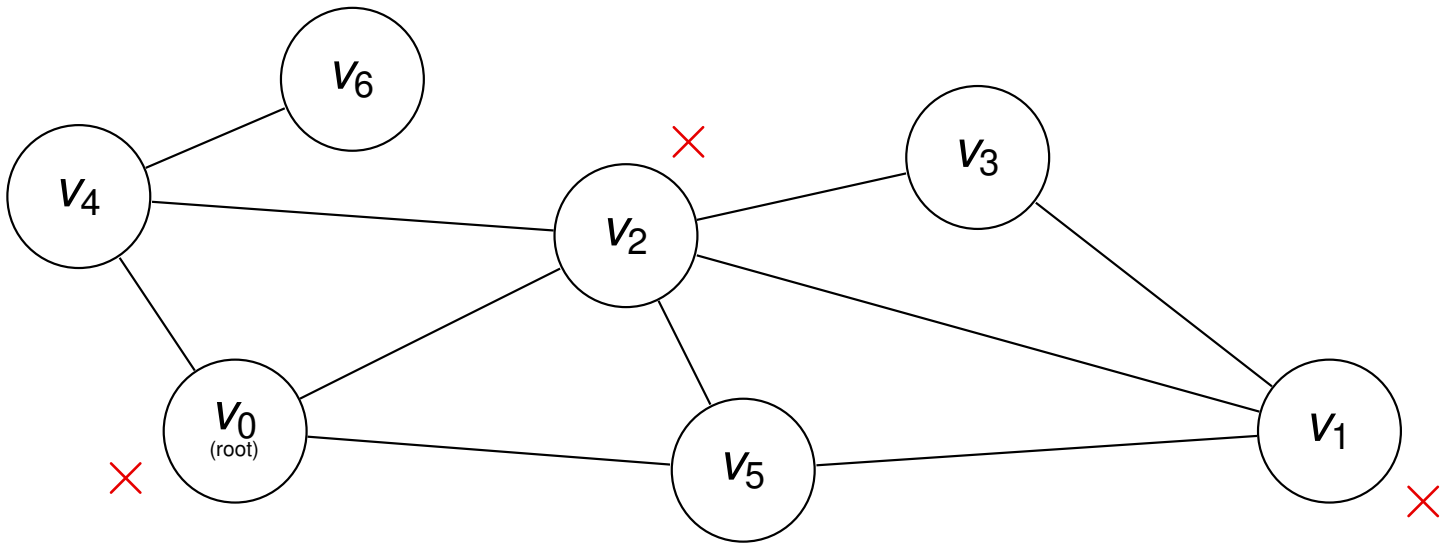


Stack:

v_5	v_4	v_5	v_4	v_3				

Output: v_0 v_2 v_1

DFS with a Stack

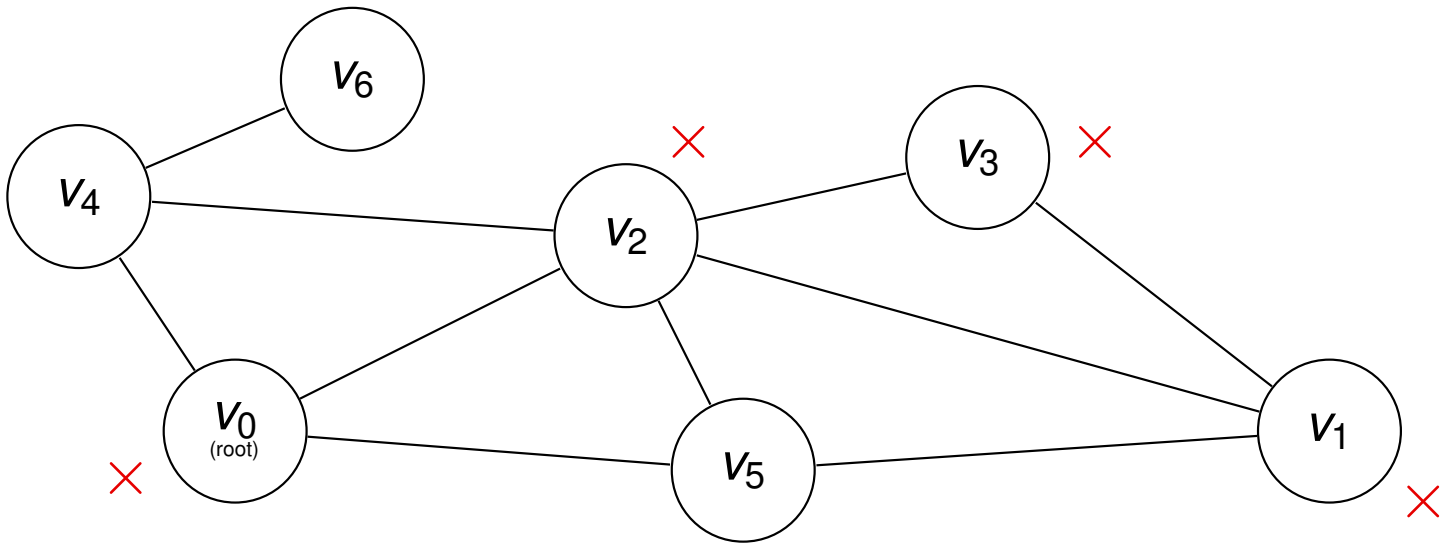


Stack:

v_5	v_4	v_5	v_4	v_3	v_5	v_3		

Output: v_0 v_2 v_1

DFS with a Stack

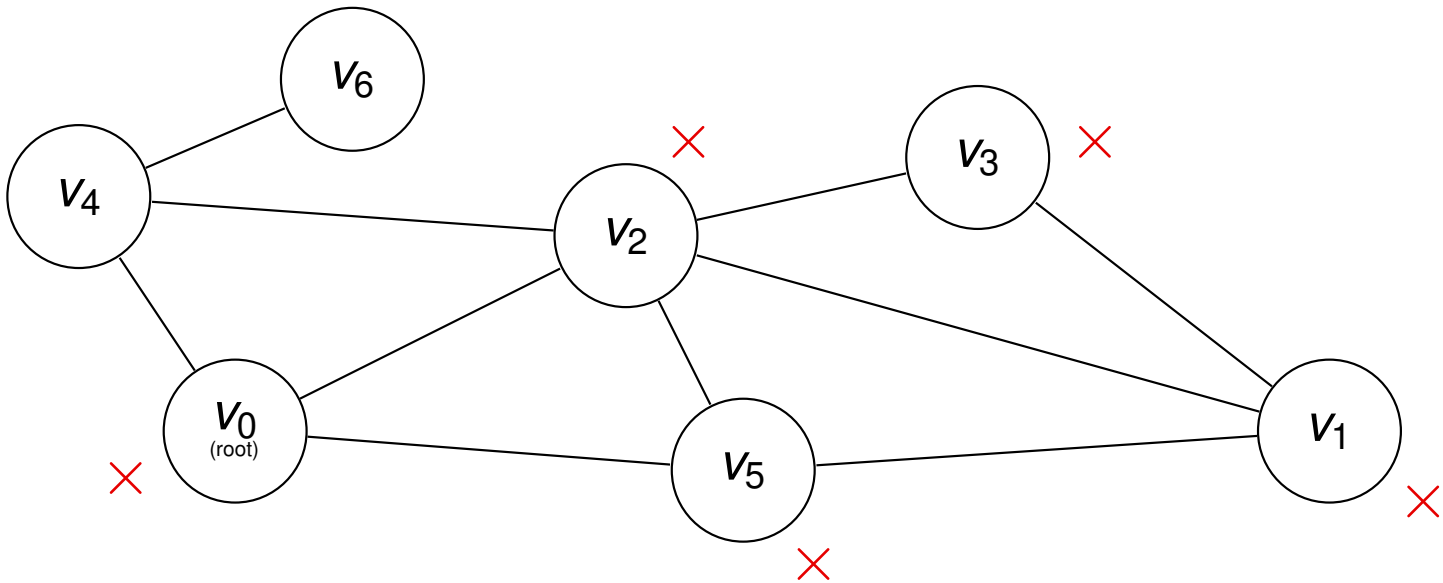


Stack:

v_5	v_4	v_5	v_4	v_3	v_5			

Output: v_0 v_2 v_1 v_3

DFS with a Stack

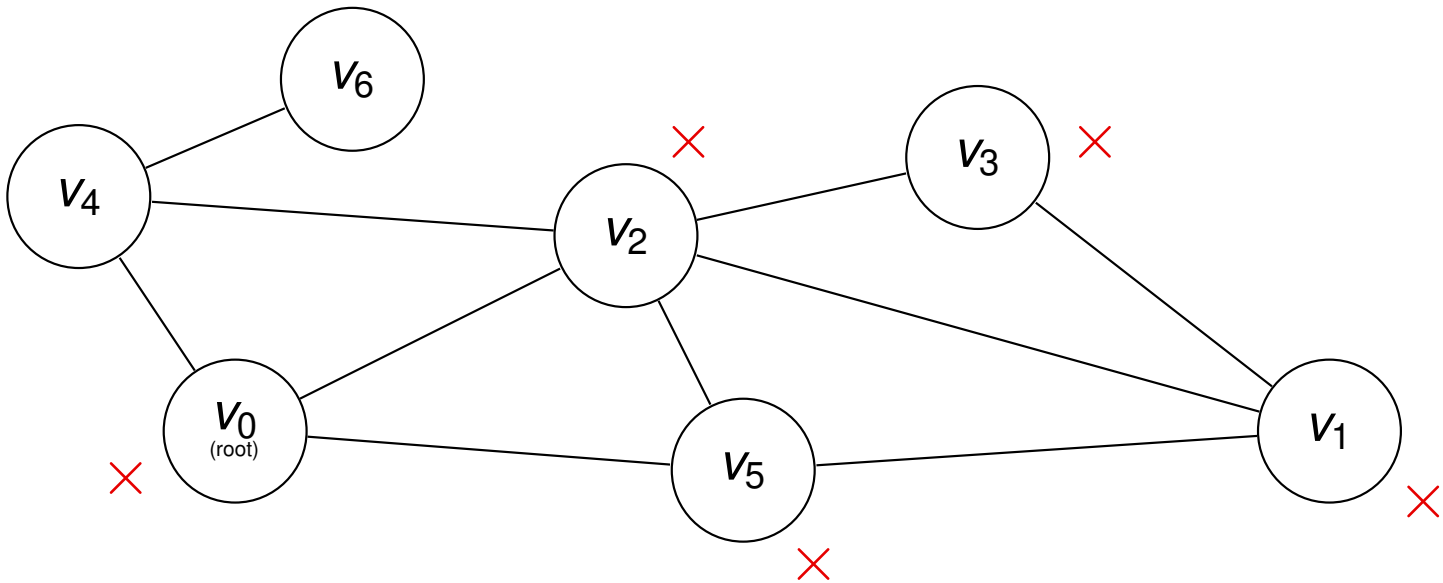


Stack:

v_5	v_4	v_5	v_4	v_3				

Output: v_0 v_2 v_1 v_3 v_5

DFS with a Stack

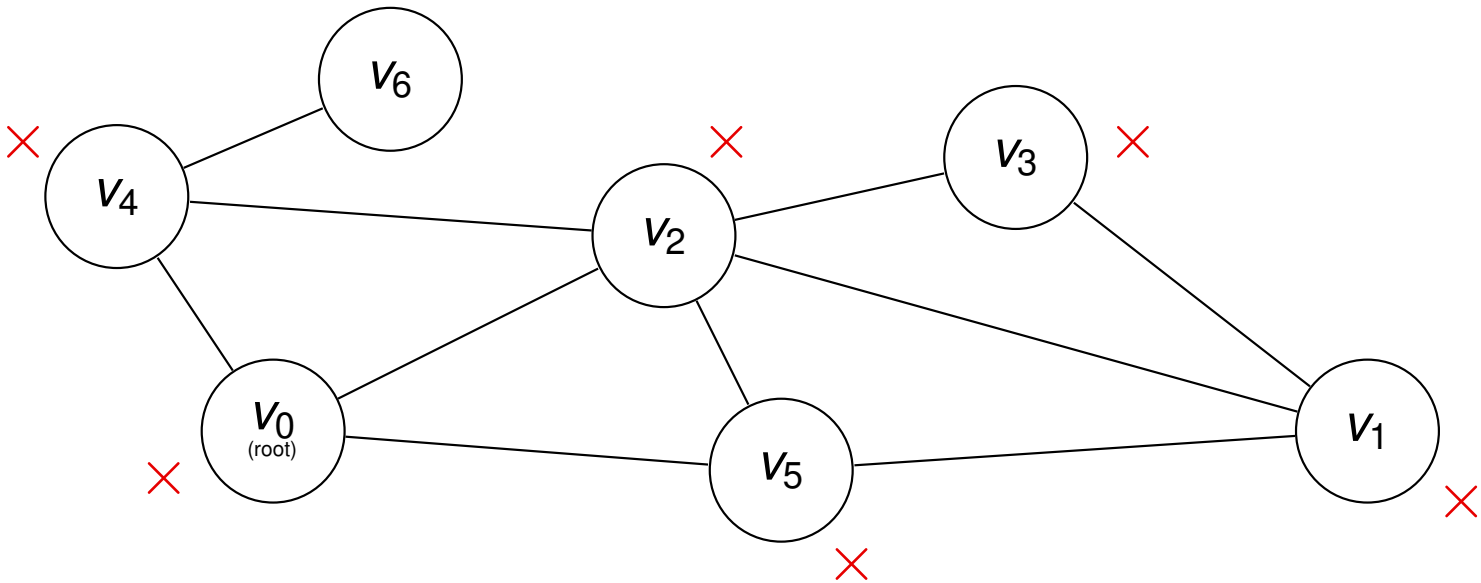


Stack:

v_5	v_4	v_5	v_4					

Output: v_0 v_2 v_1 v_3 v_5

DFS with a Stack

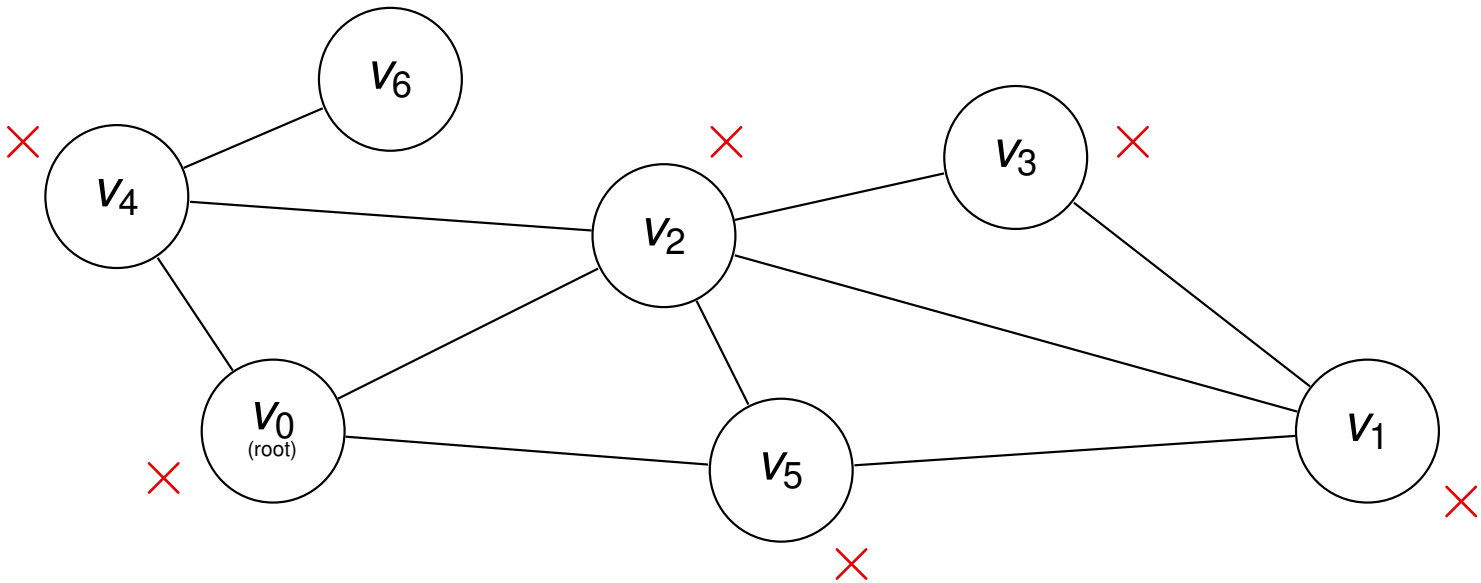


Stack:

v_5	v_4	v_5						

Output: v_0 v_2 v_1 v_3 v_5 v_4

DFS with a Stack

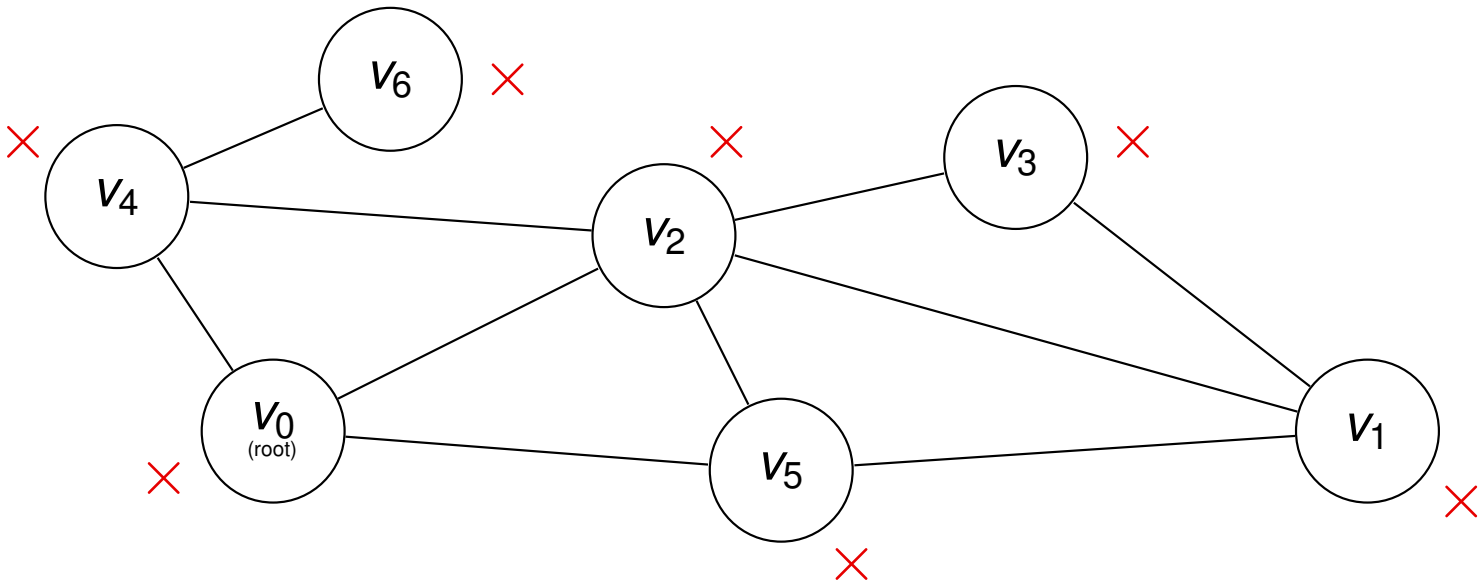


Stack:

v_5	v_4	v_5	v_6					

Output: v_0 v_2 v_1 v_3 v_5 v_4

DFS with a Stack

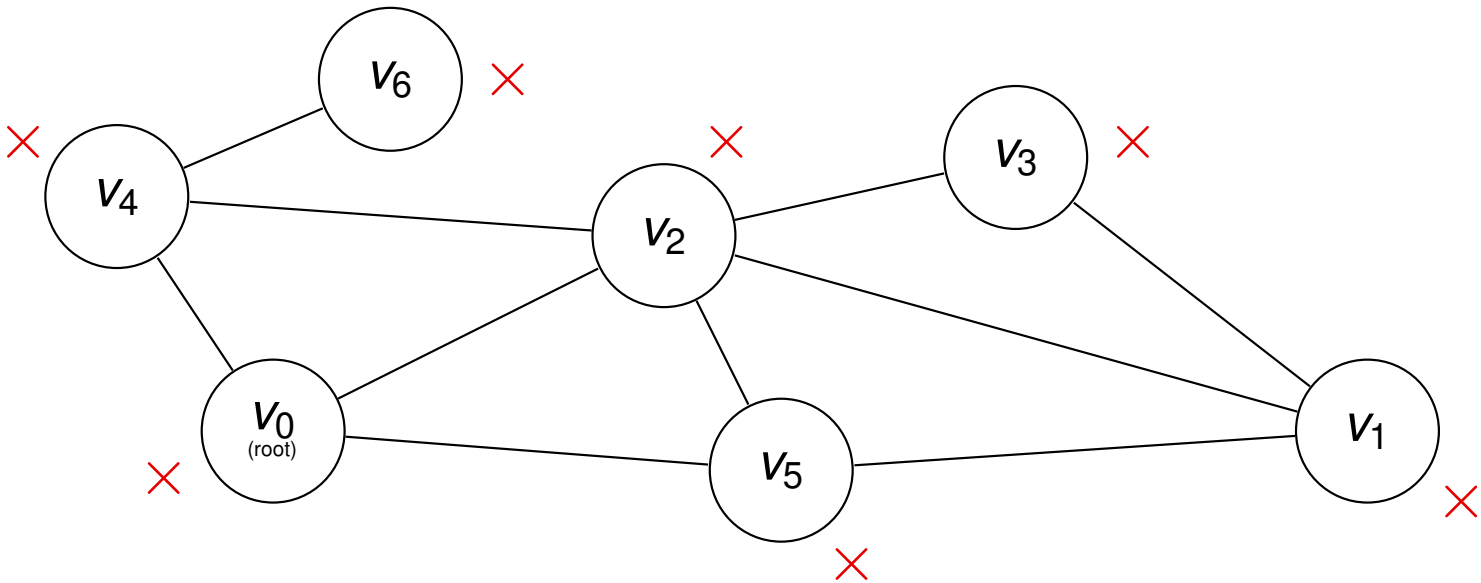


Stack:

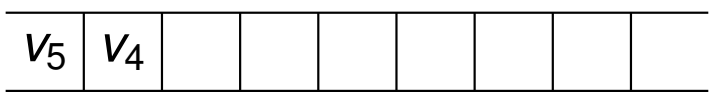
v_5	v_4	v_5						

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS with a Stack

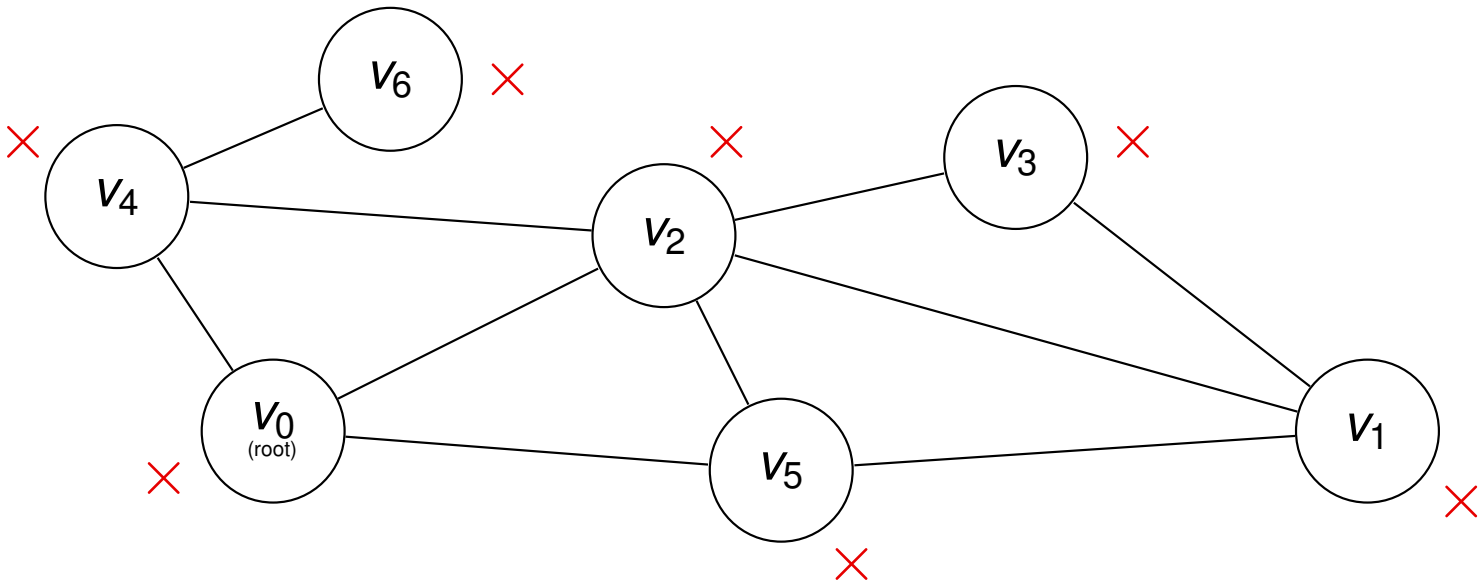


Stack:

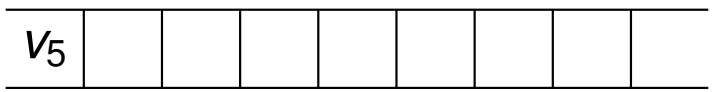


Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS with a Stack

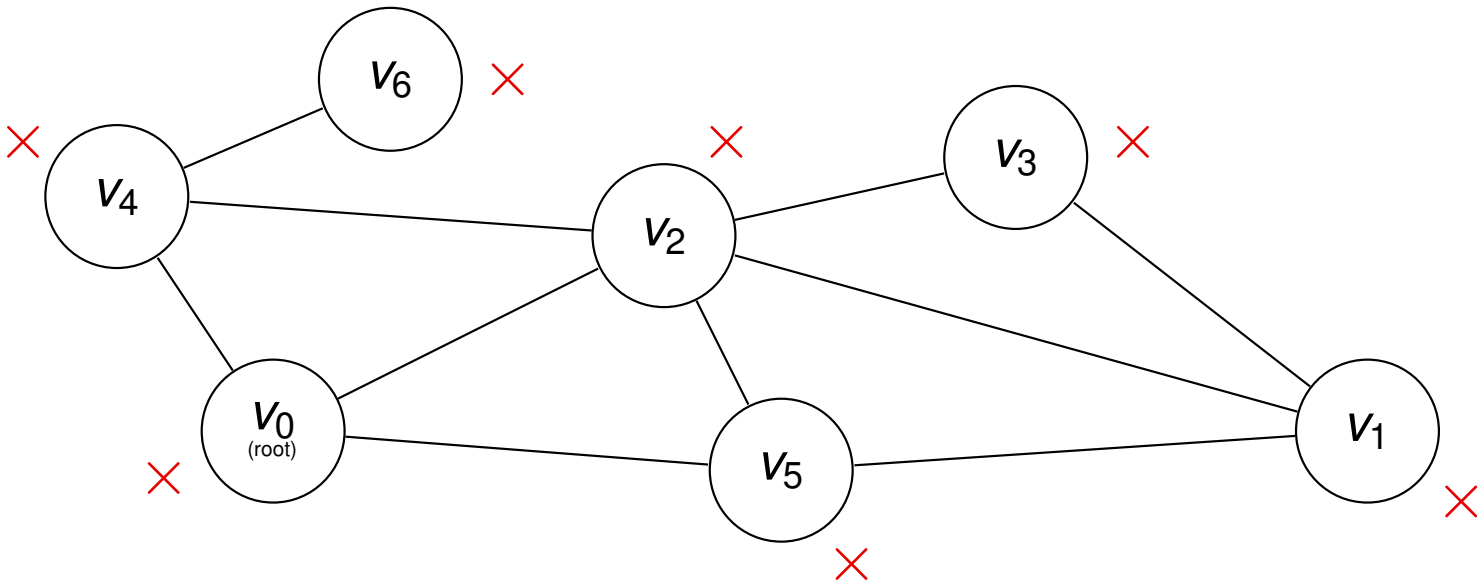


Stack:

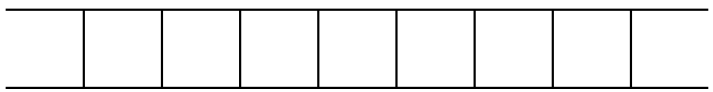


Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS with a Stack



Stack:



Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS with a Stack

```
void dfs() {
   stack<int> vertices;
   vertices.push(0);
   int current, v;
   while (vertices.size() > 0) {
      current = vertices.top();
      vertices.pop();
      if (!visited[current]) {
         visited[current] = true;
         cout << current << " ";
         for (int i=G[current].size()-1; i>=0; --i) {
            v = G[current][i];
            vertices.push(v);
         }
      }
   }
}
```

Depth-First Search

in undirected graphs recursively

Recursive DFS

- Again start at root and mark it as visited

Recursive DFS

- Again start at root and mark it as visited
- Recursively call algorithm on non-visited neighbors

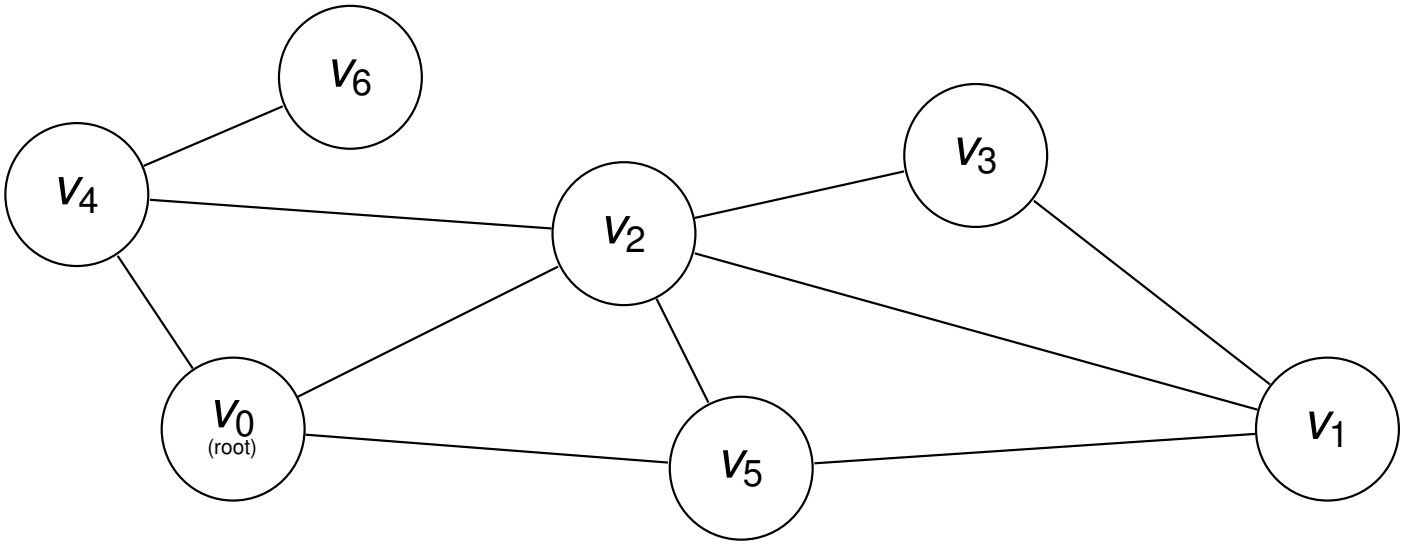
Recursive DFS

- Again start at root and mark it as visited
- Recursively call algorithm on non-visited neighbors

```
vector<vector<int>> G {{1,2},{2},{0}};
vector<int> visited(G.size(),false);

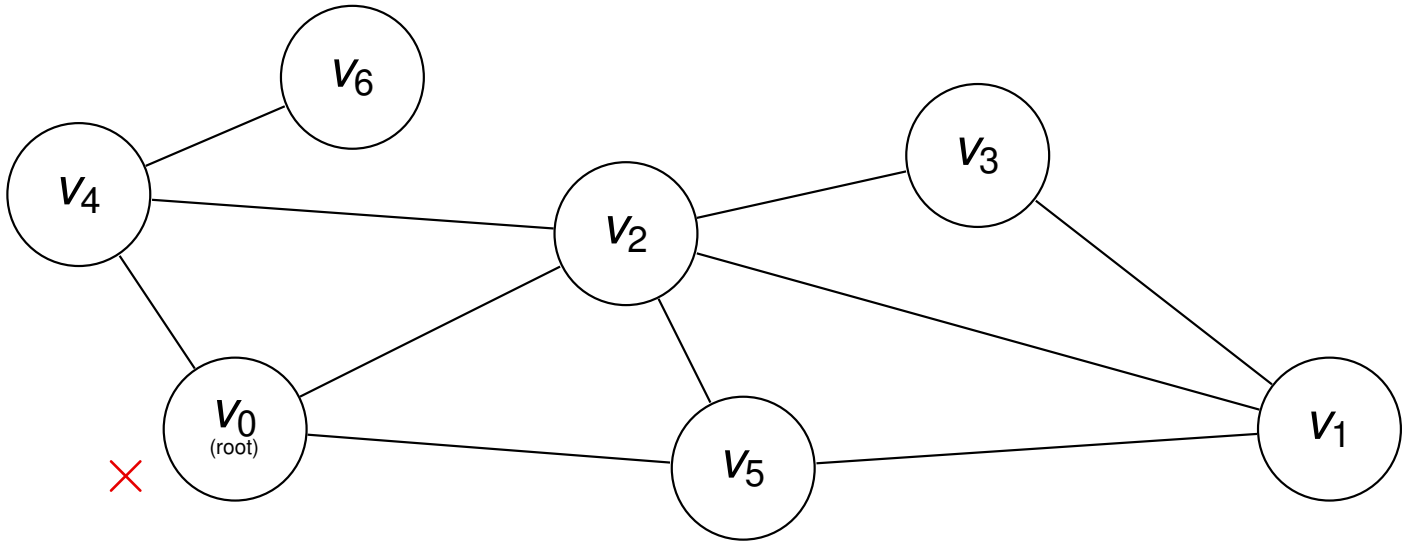
void dfs(int current) {
   if (!visited[current]) {
      visited[current] = true;
      cout << current << " ";
      for (int i=0; i<G[current].size(); ++i) {
         int v = G[current][i];
         dfs(v);
      }
   }
}
```

Recursive DFS



Output:

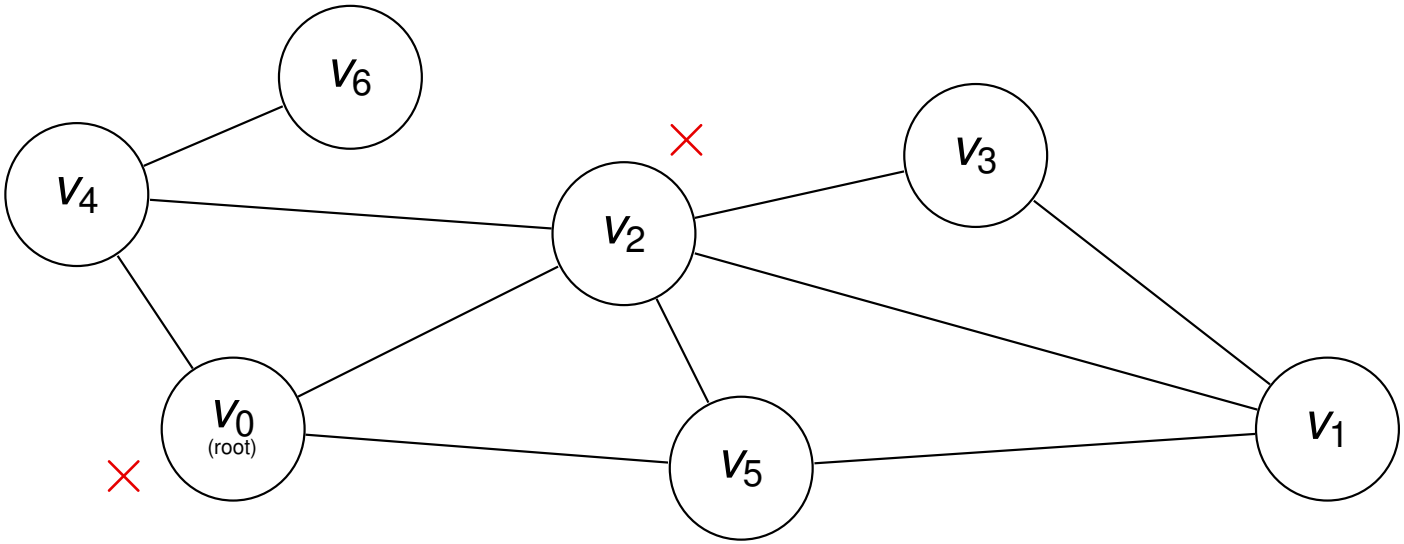
Recursive DFS




```
dfs(0)
```

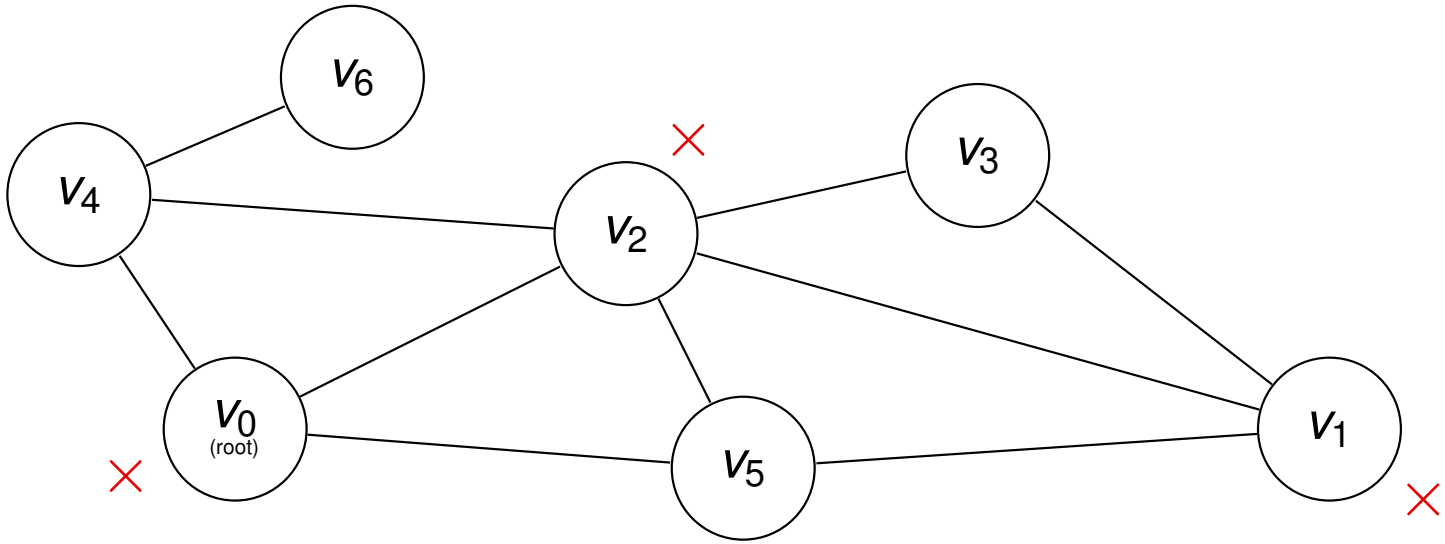
Output: v_0

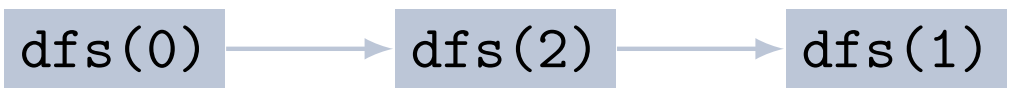
Recursive DFS



Output: v_0 v_2

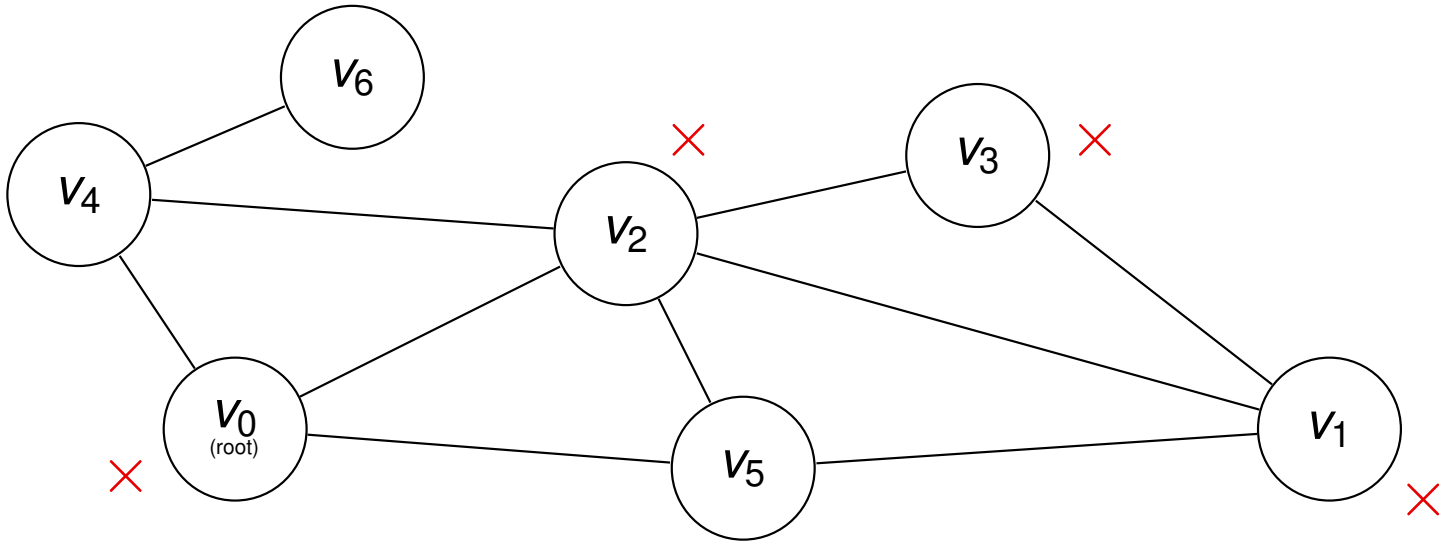
Recursive DFS





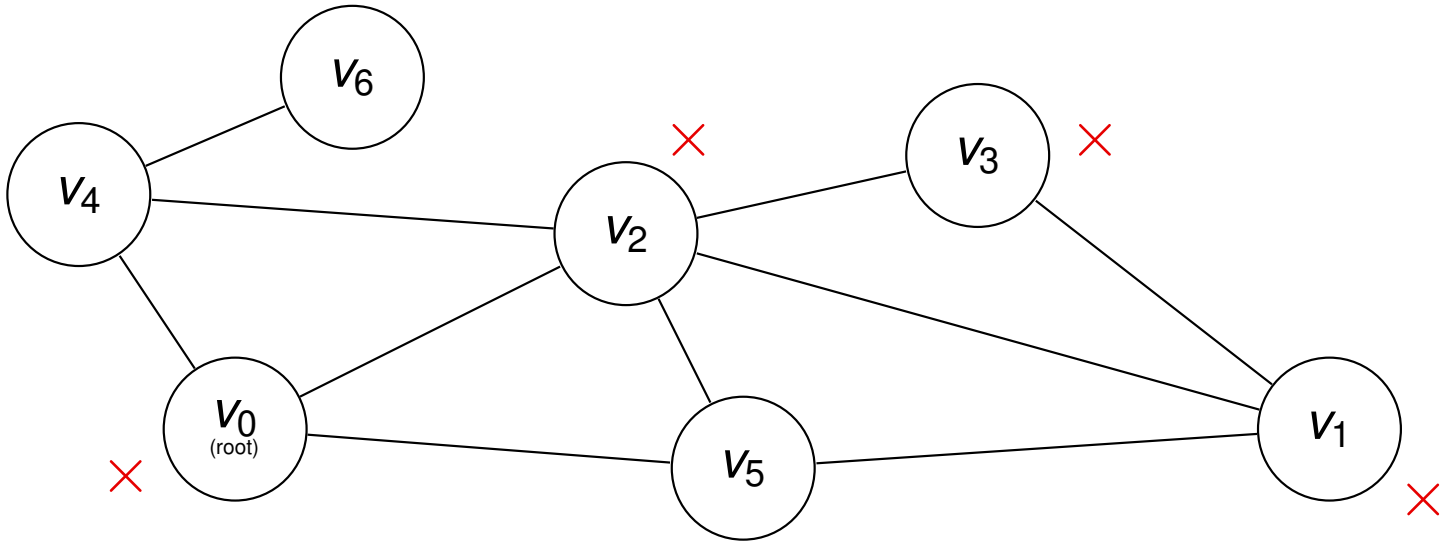
Output: v_0 v_2 v_1

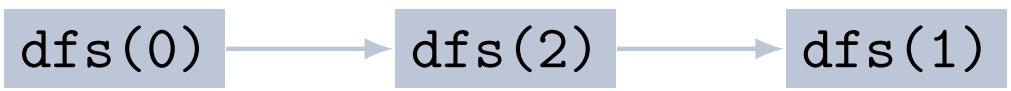
Recursive DFS



Output: v_0 v_2 v_1 v_3

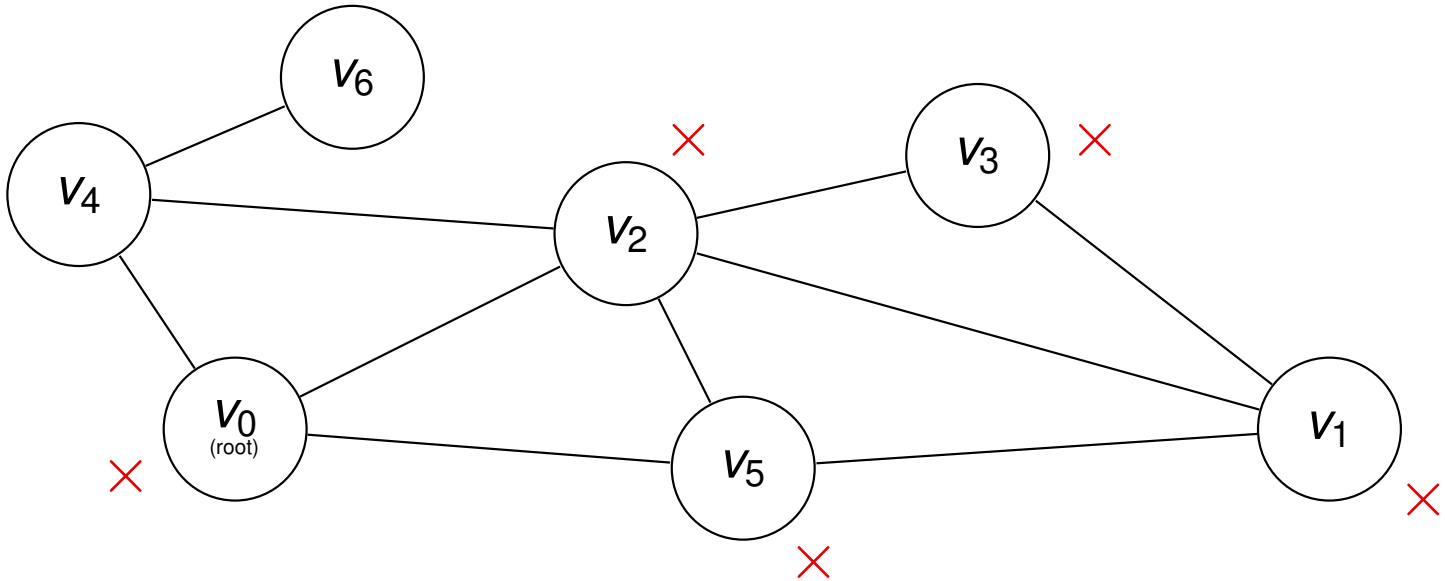
Recursive DFS





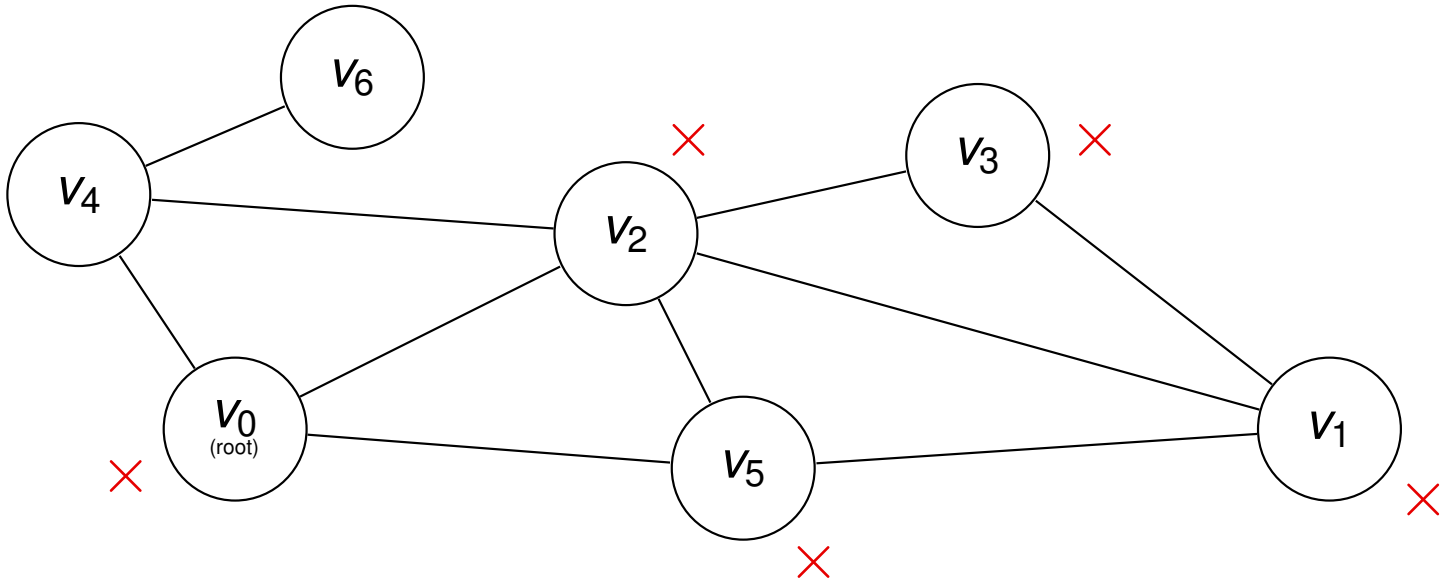
Output: v_0 v_2 v_1 v_3

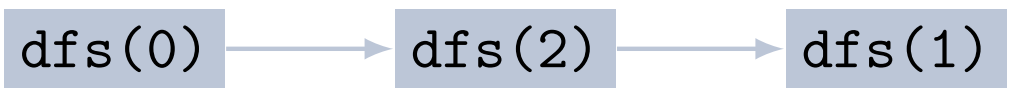
Recursive DFS



Output: v_0 v_2 v_1 v_3 v_5

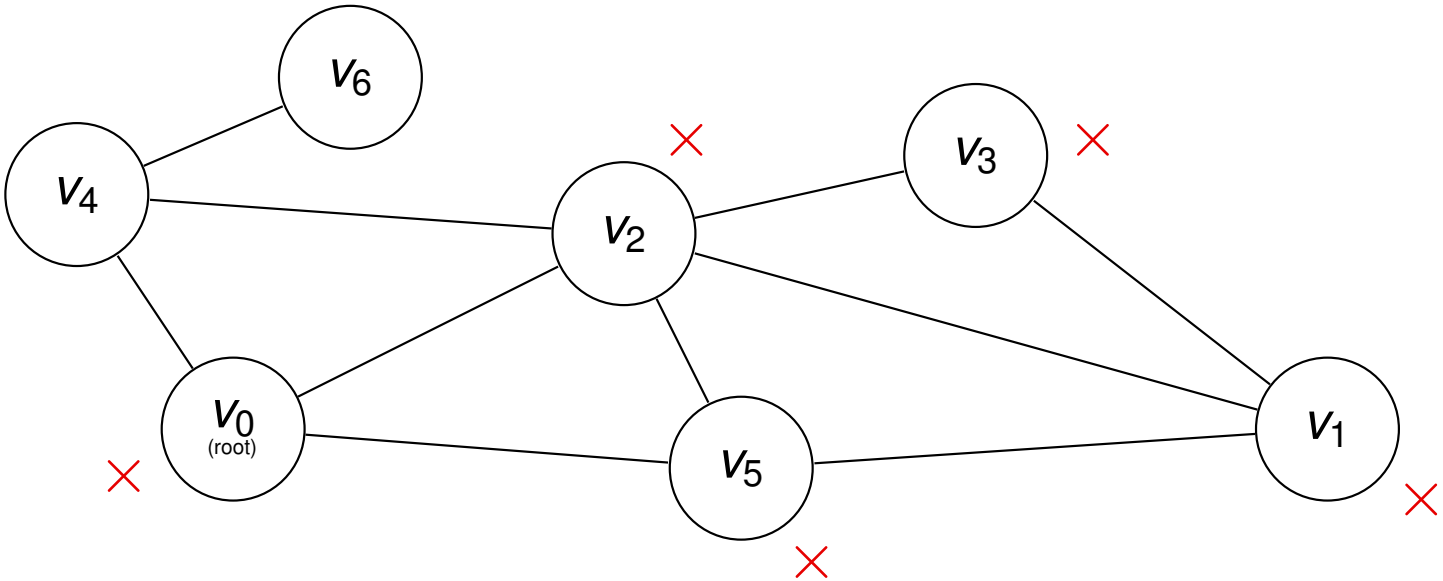
Recursive DFS





Output: v_0 v_2 v_1 v_3 v_5

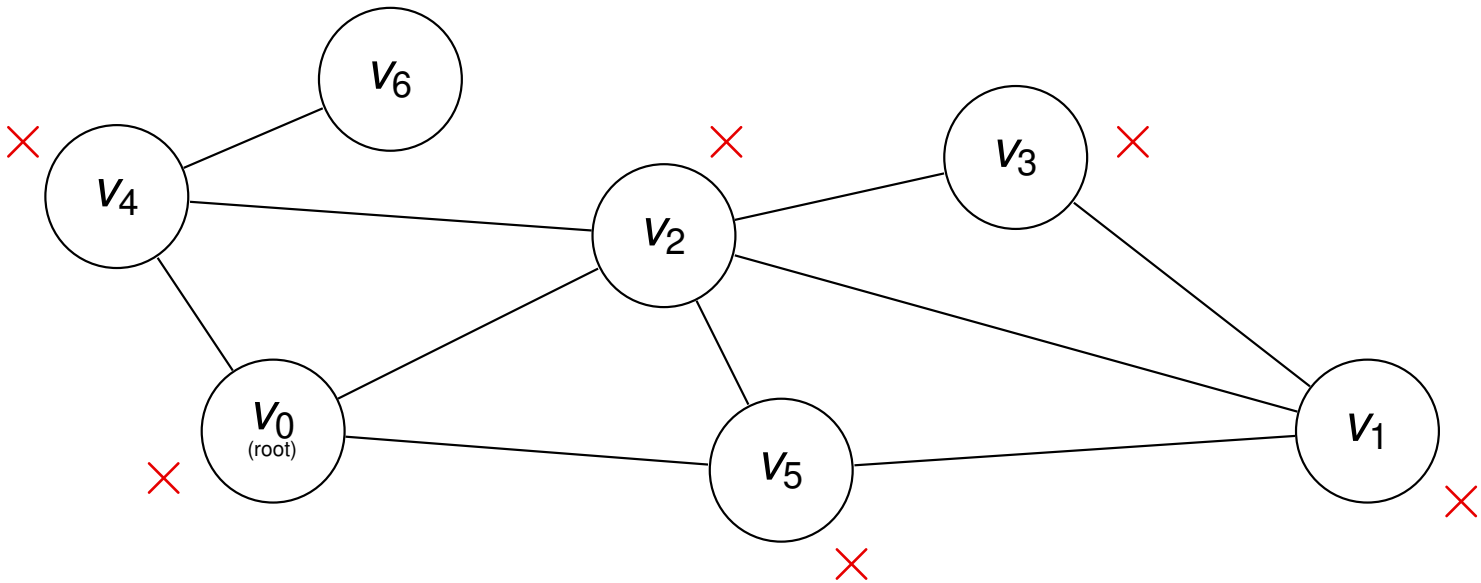
Recursive DFS

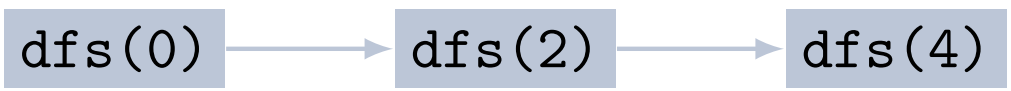



```
dfs(0) → dfs(2)
```

Output: v_0 v_2 v_1 v_3 v_5

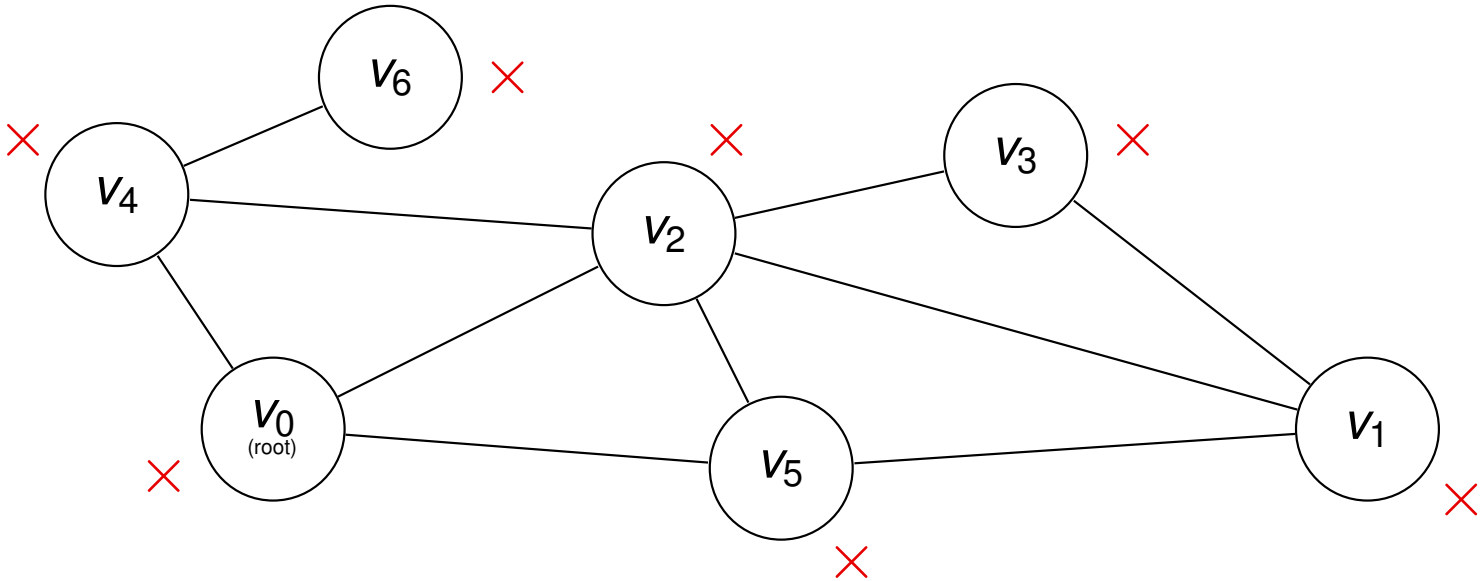
Recursive DFS





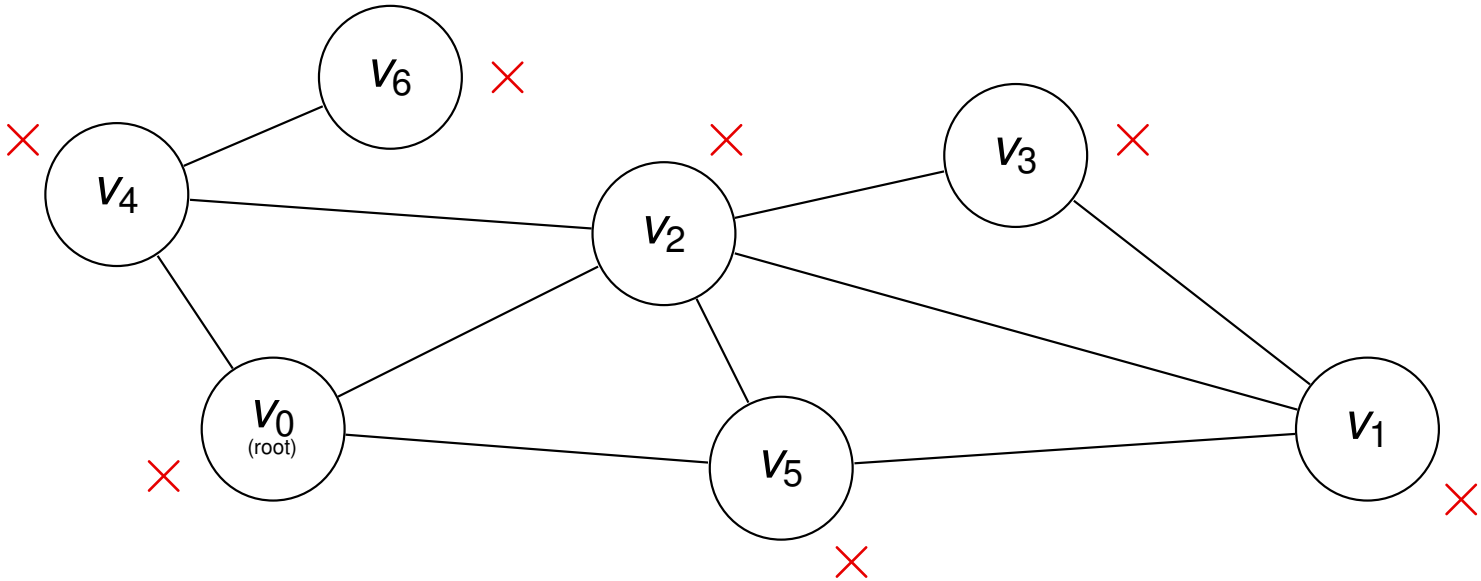
Output: v_0 v_2 v_1 v_3 v_5 v_4

Recursive DFS



Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

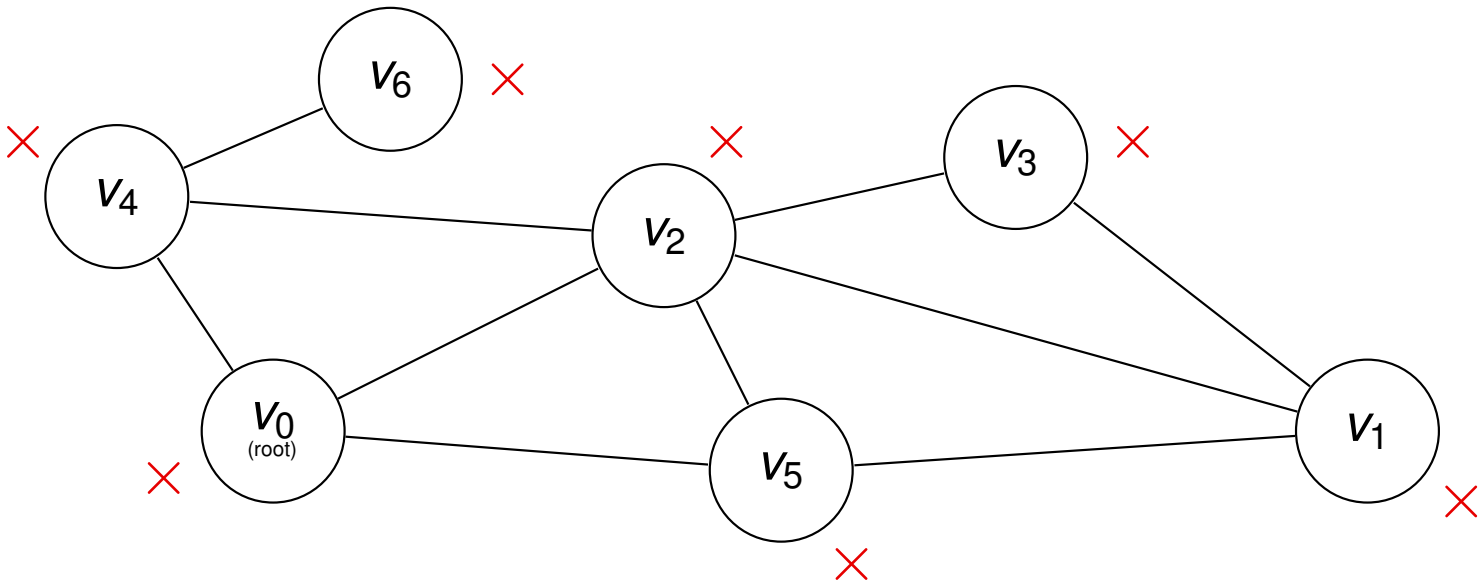
Recursive DFS




```
dfs(0) → dfs(2) → dfs(4)
```

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

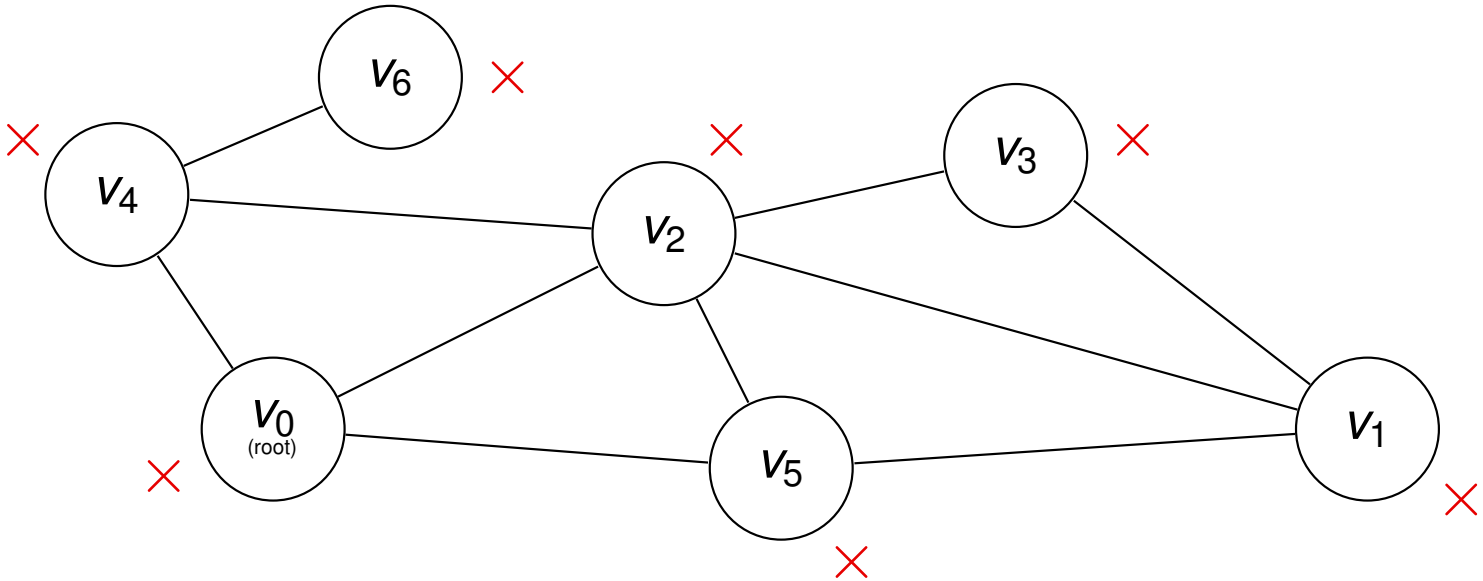
Recursive DFS




```
dfs(0) → dfs(2)
```

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

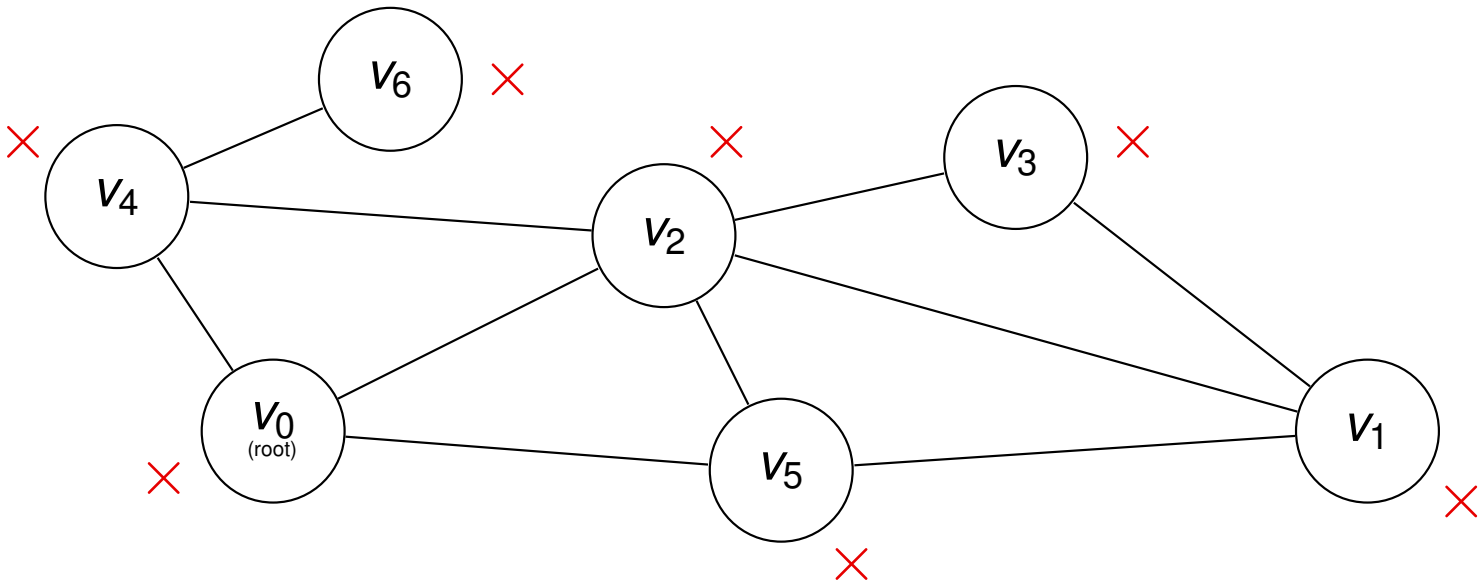
Recursive DFS




```
dfs(0)
```

Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

Recursive DFS

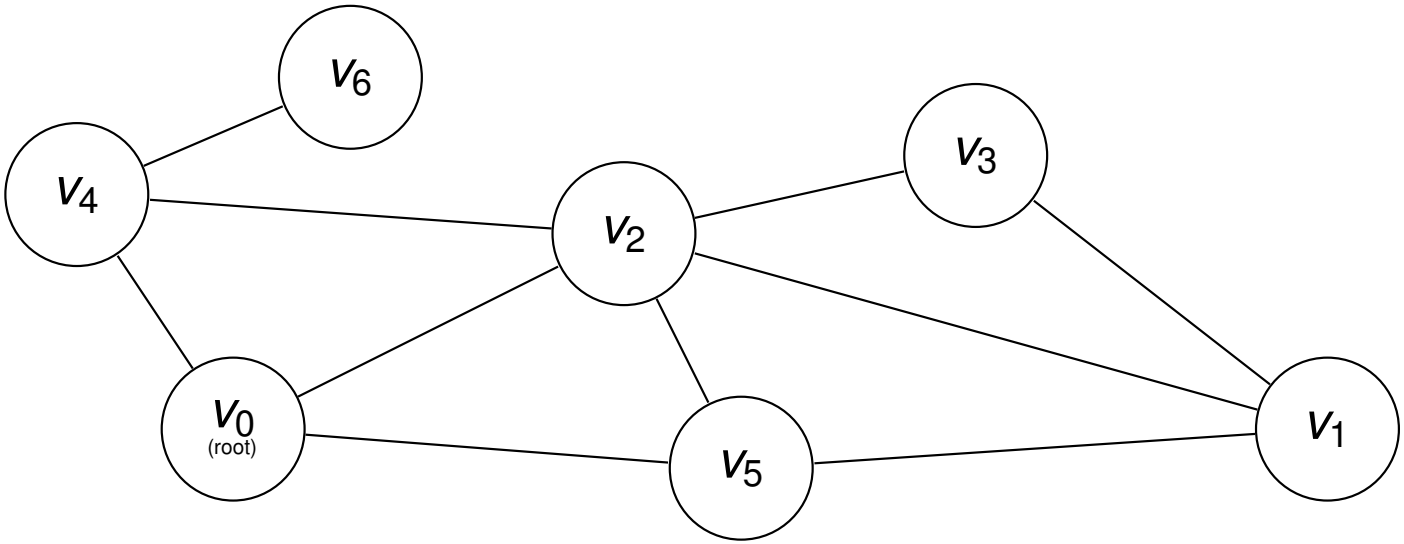


Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

DFS Tree

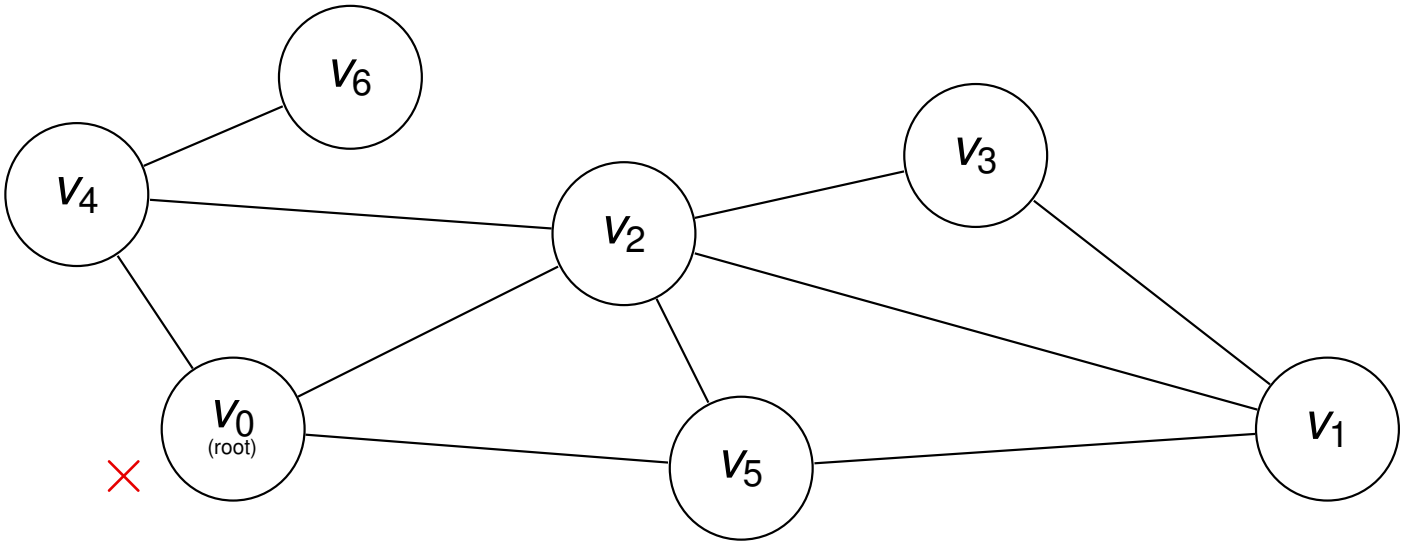
of an undirected graph

DFS Tree



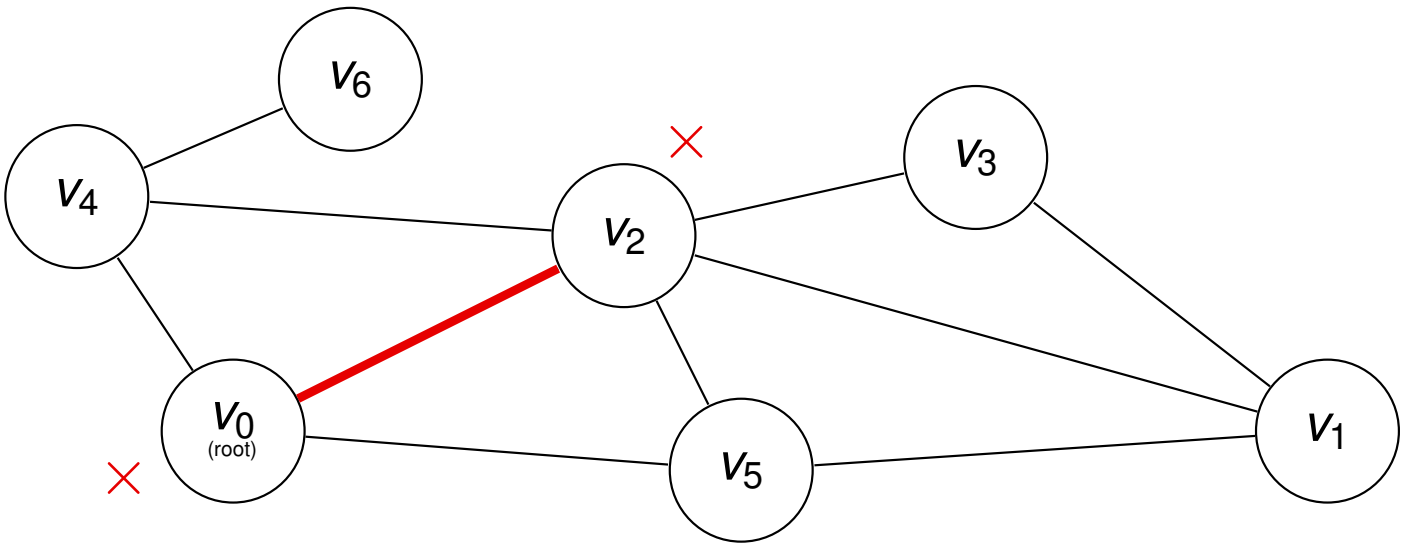
Output:

DFS Tree



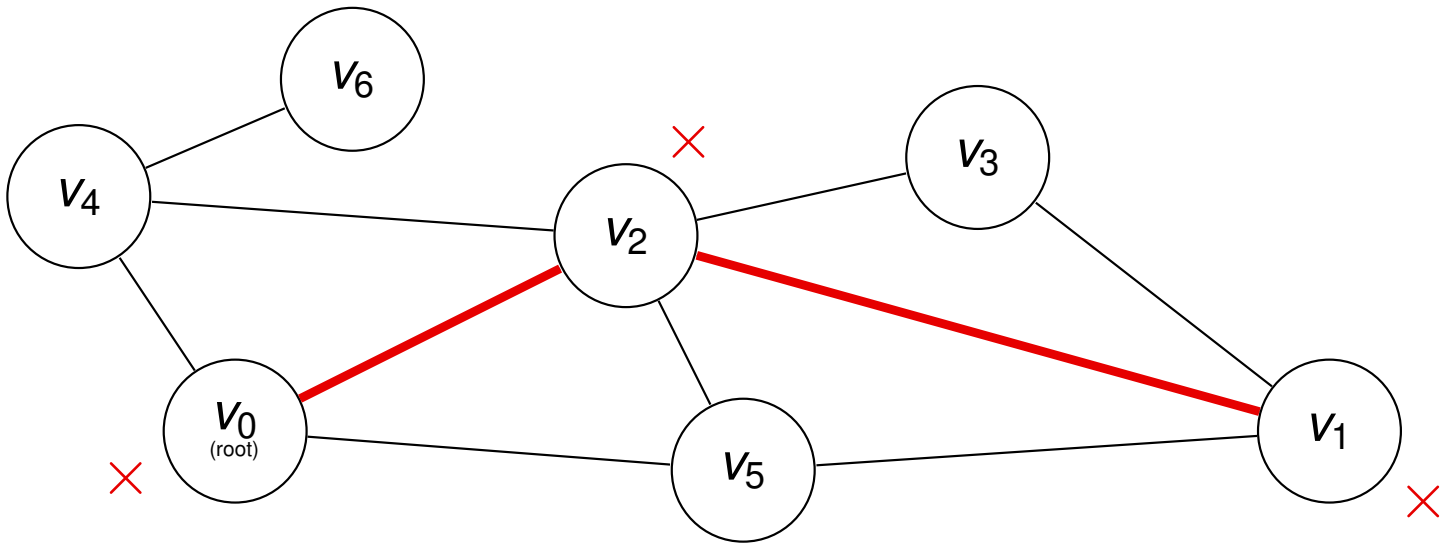
Output: v_0

DFS Tree



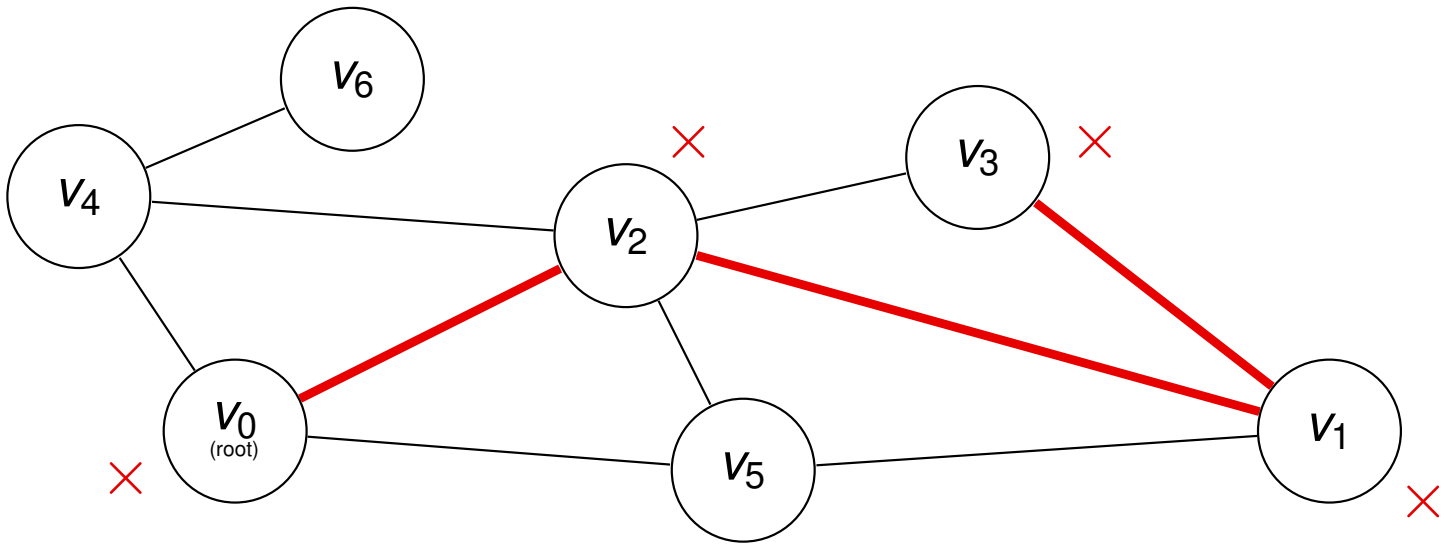
Output: v_0 v_2

DFS Tree



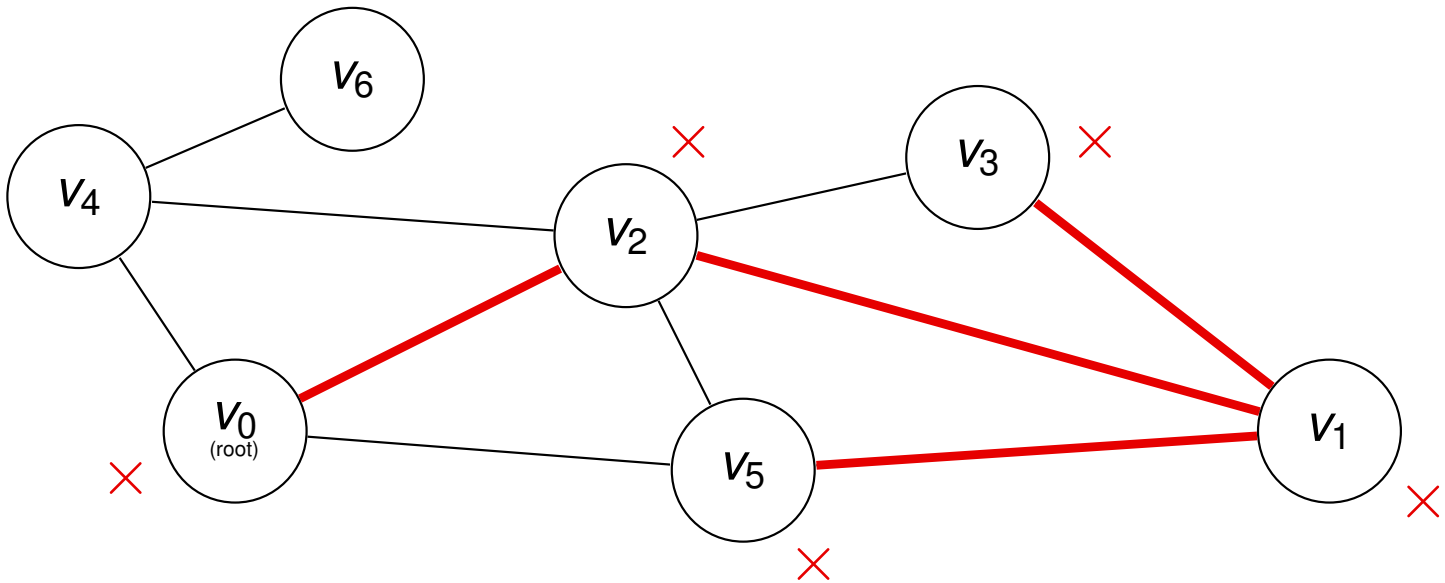
Output: v_0 v_2 v_1

DFS Tree



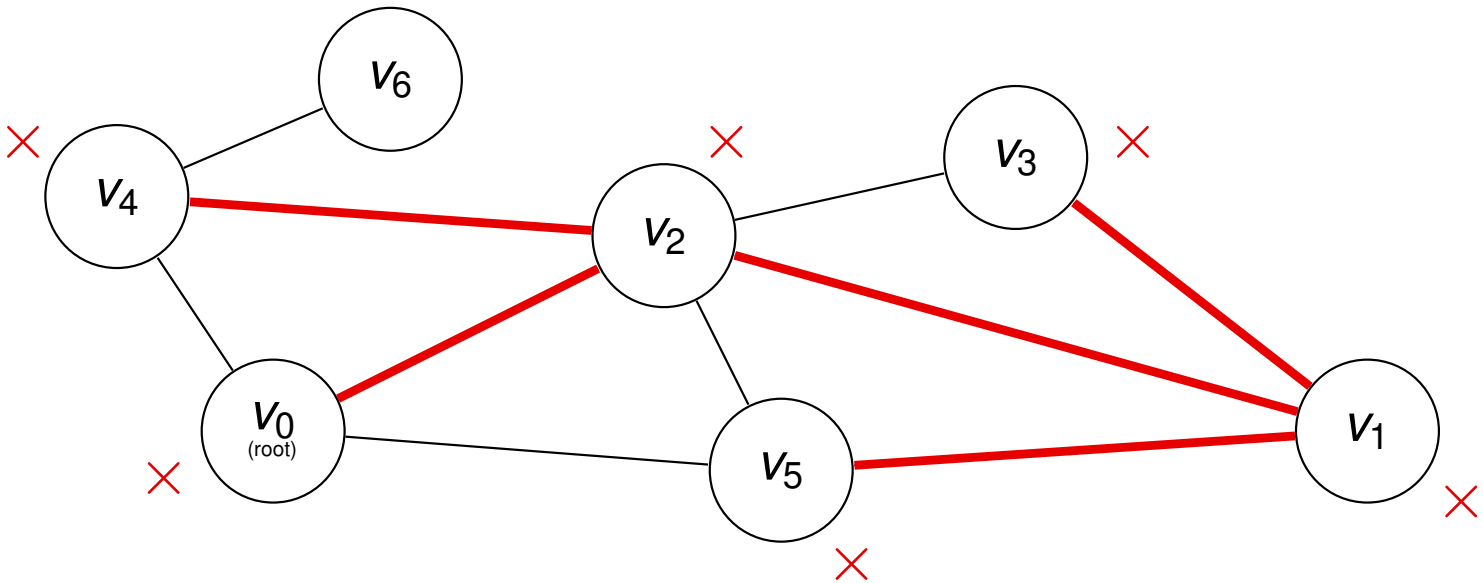
Output: v_0 v_2 v_1 v_3

DFS Tree



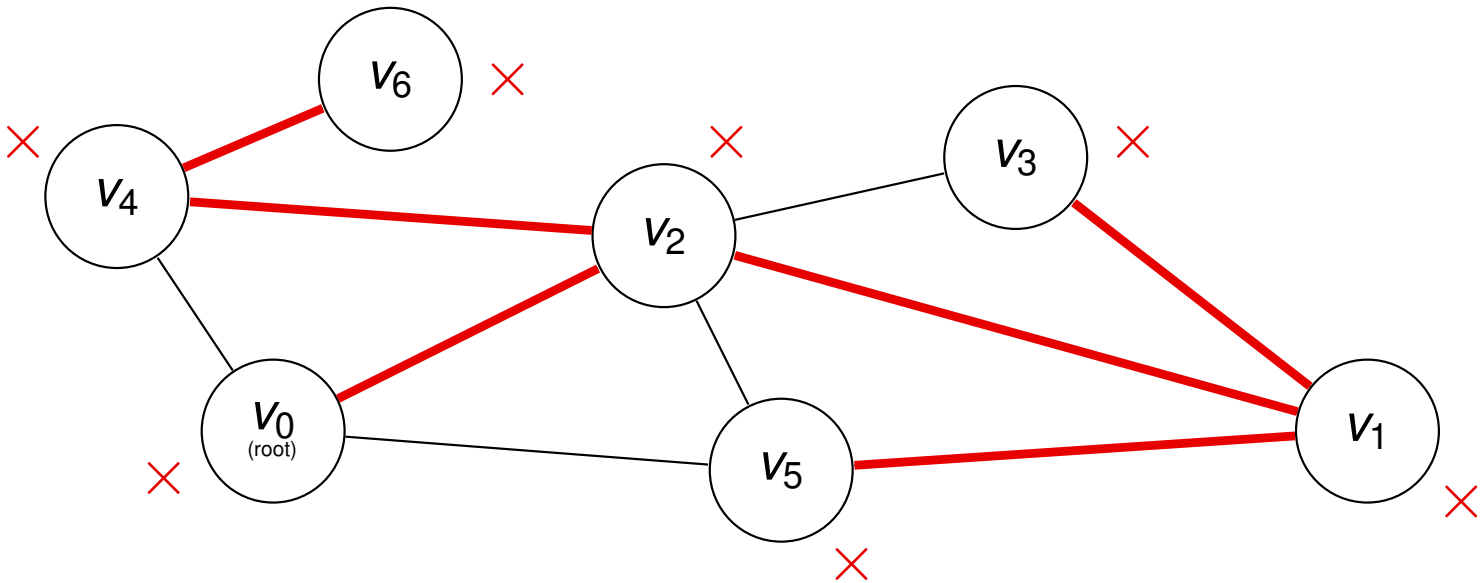
Output: v_0 v_2 v_1 v_3 v_5

DFS Tree



Output: v_0 v_2 v_1 v_3 v_5 v_4

DFS Tree

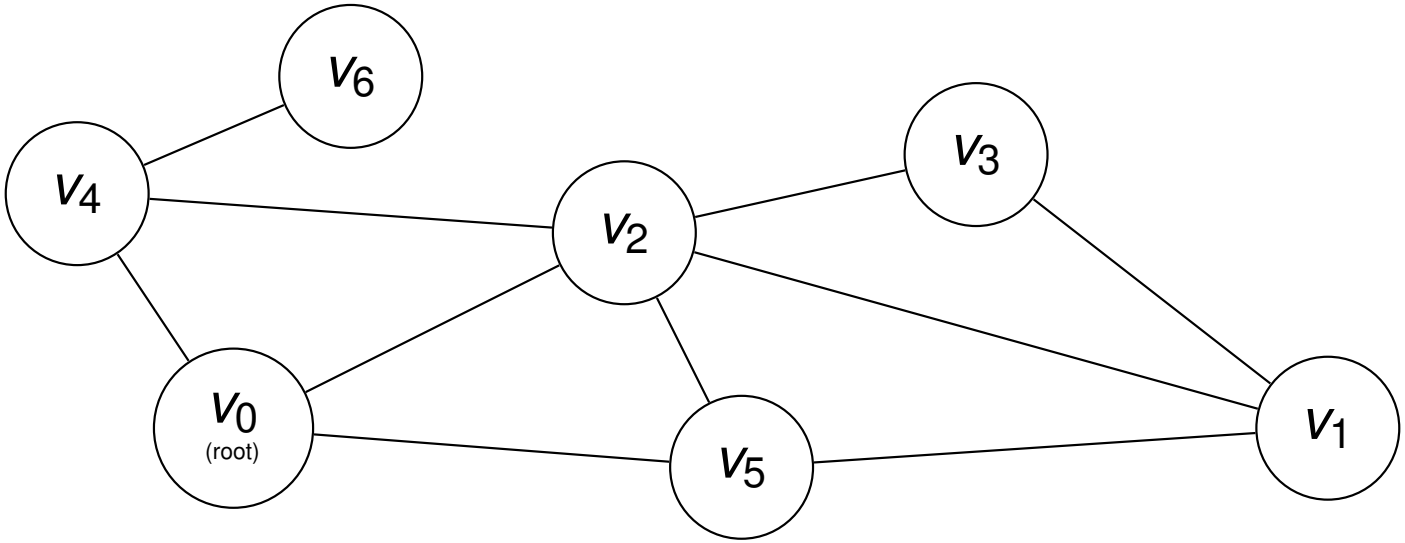


Output: v_0 v_2 v_1 v_3 v_5 v_4 v_6

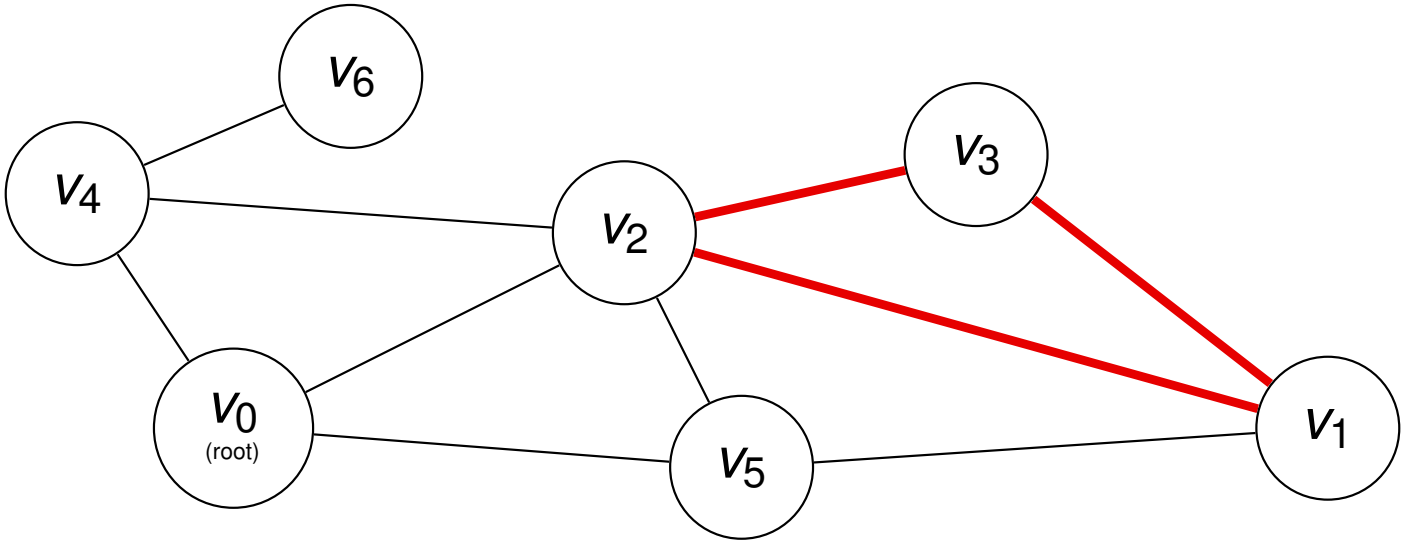
Finding Cycles

in undirected graphs

Finding Cycles

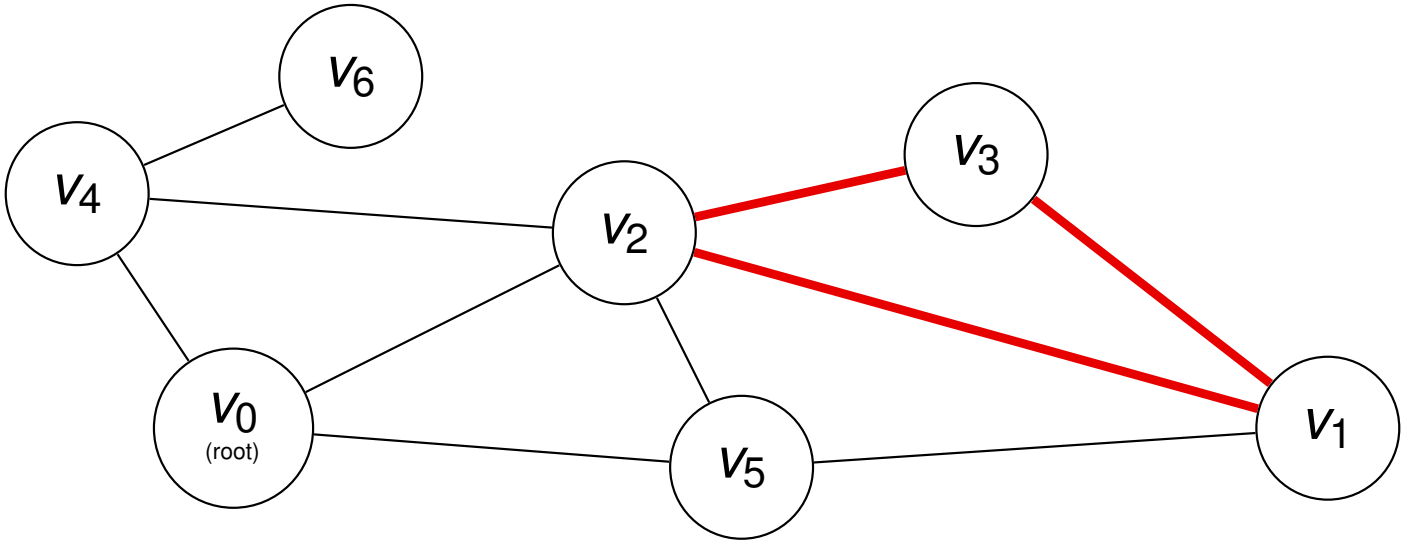


Finding Cycles



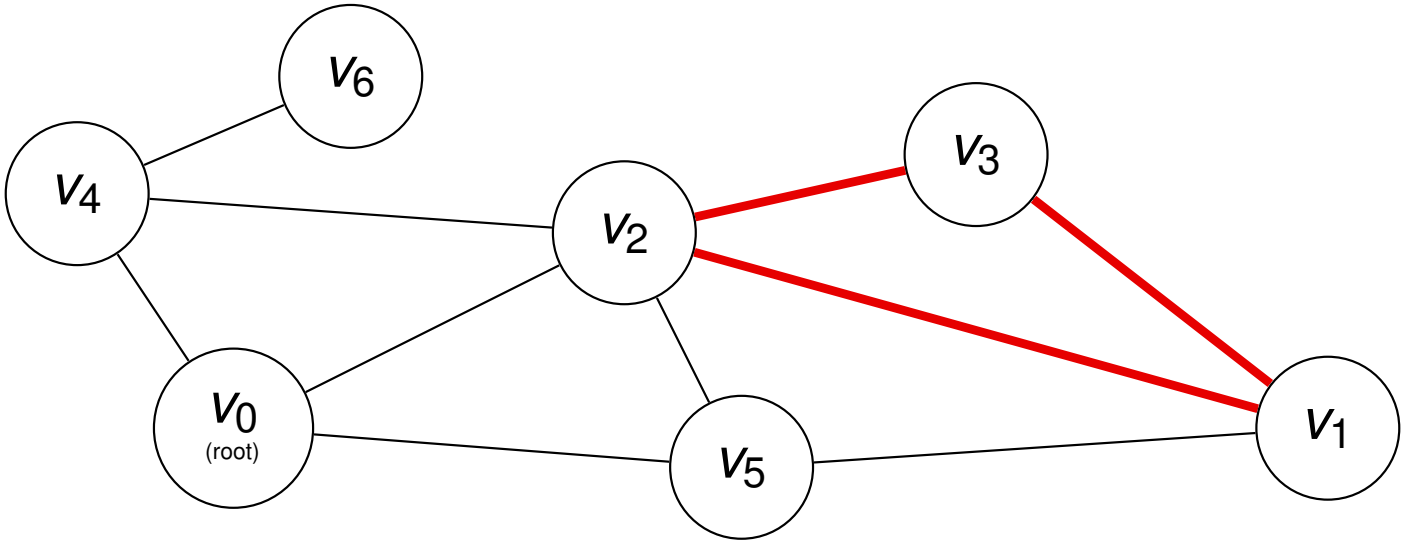
- DFS supplies all we need

Finding Cycles



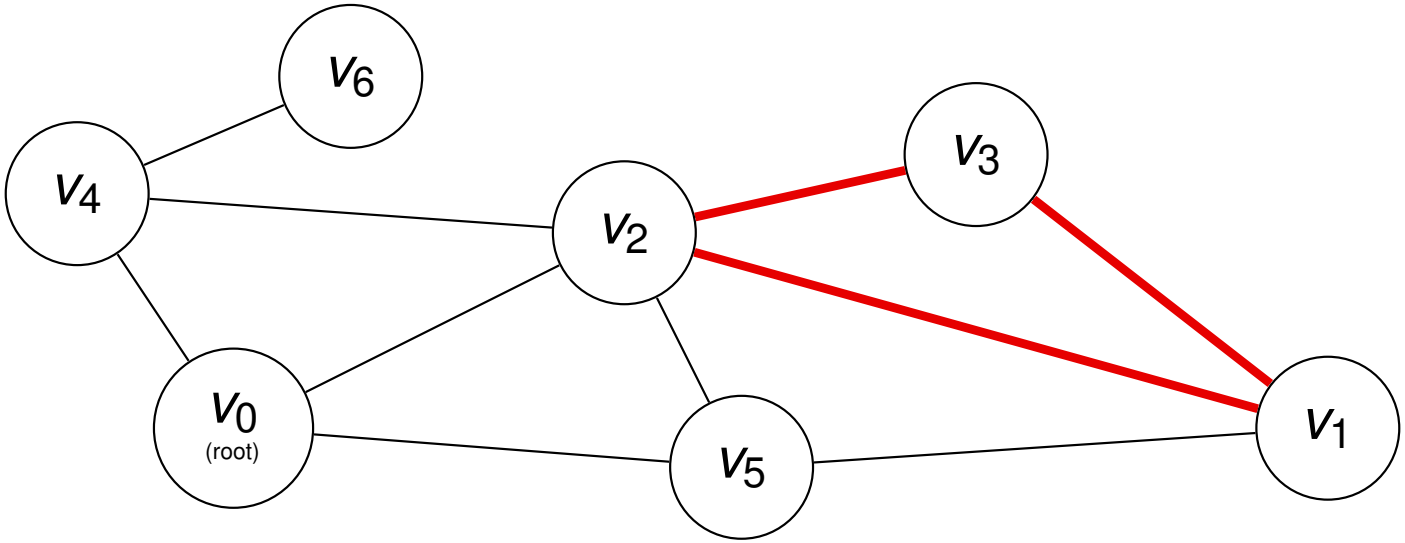
- DFS supplies all we need
- Traverse graph as before

Finding Cycles



- DFS supplies all we need
- Traverse graph as before
- Find edge connecting current vertex to one already visited?

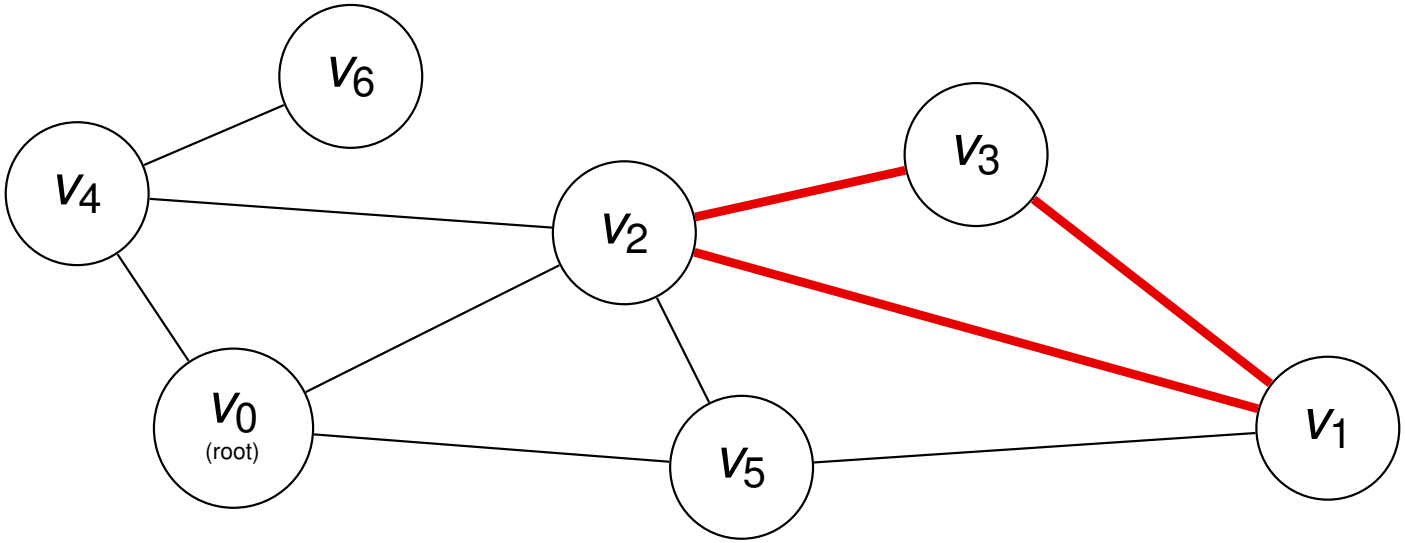
Finding Cycles



- DFS supplies all we need
- Traverse graph as before
- Find edge connecting current vertex to one already visited?

⇨ Back edge

Finding Cycles



- DFS supplies all we need
- Traverse graph as before
- Find edge connecting current vertex to one already visited?

⇨ Back edge

- Careful though: A single edge is not a cycle

Finding Cycles

```
vector<vector<int>>
G { {2,4,5},
    {2,3,5},
    {0,1,3,4,5},
    {1,2},
    {0,2,6},
    {0,1,2},
    {4} };
```

Finding Cycles

Extend DFS to keep track of parent vertex

Finding Cycles

Extend DFS to keep track of parent vertex

```
void cyclefind(int current, int parent) {
   if (!visited[current]) {
      cout << current << " ";
      visited[current] = true;
      for (int i=0; i<G[current].size(); ++i) {
         int v = G[current][i];
         if (v != parent) {
            if (visited[v]) {
               cout << "CYCLE";
            } else {
               cyclefind(v,current);
            }
         }
      }
   }
}
```

Other Applications

DFS in undirected graphs

Other Applications

Is graph connected?

Other Applications

Is graph connected?

- DFS from arbitrary vertex

Other Applications

Is graph connected?

- DFS from arbitrary vertex
- Are all vertices visited when done?

Other Applications

Is graph connected?

- DFS from arbitrary vertex
- Are all vertices visited when done?

Is vertex w reachable from vertex v?

Other Applications

Is graph connected?

- DFS from arbitrary vertex
- Are all vertices visited when done?

Is vertex w reachable from vertex v?

- DFS from v

Other Applications

Is graph connected?

- DFS from arbitrary vertex
- Are all vertices visited when done?

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Finding Cycles

in directed graphs

Finding Cycles

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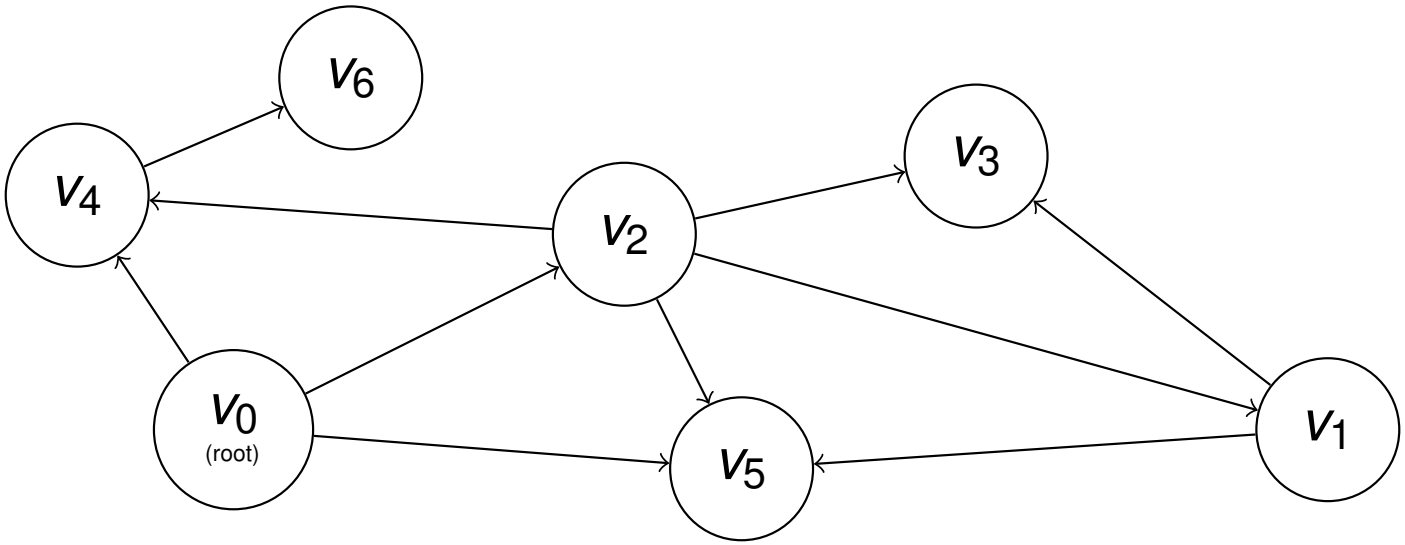

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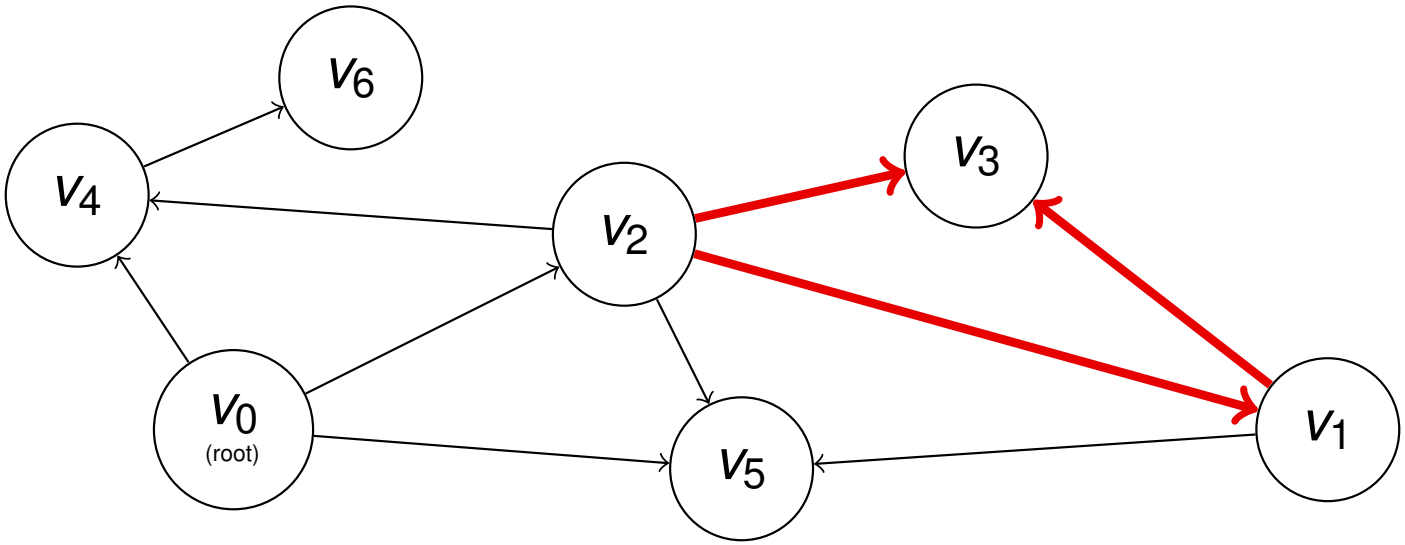

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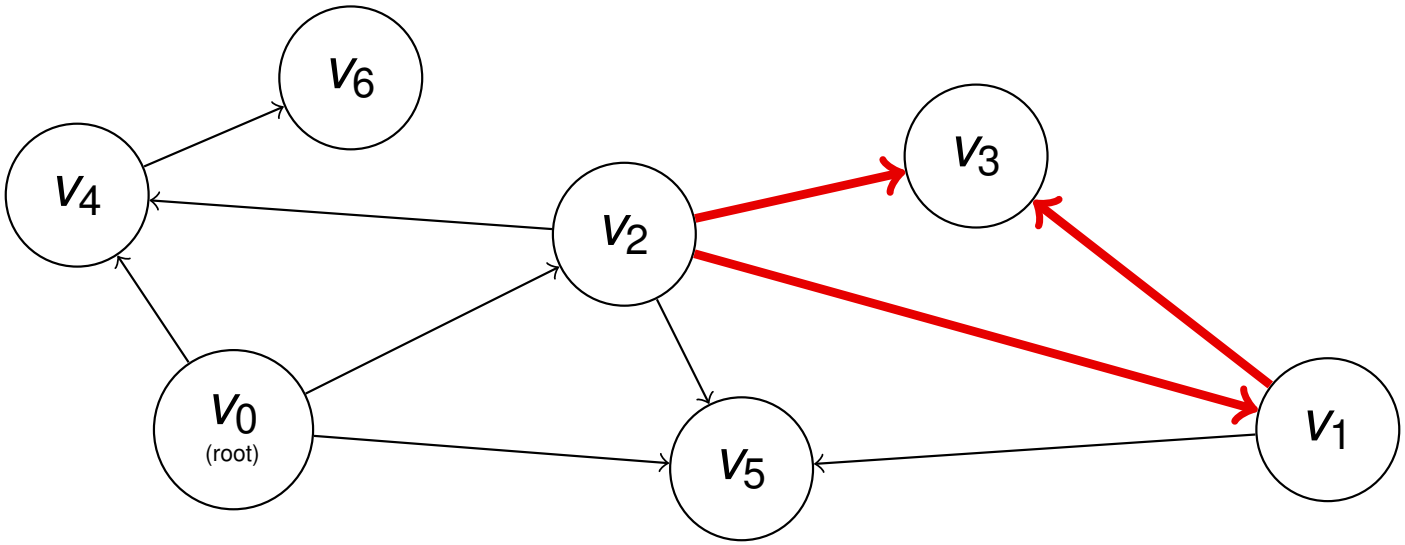

- This is not a cycle
- But our algorithm would identify it as one

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- unvisited (marked white)
- active (marked gray)
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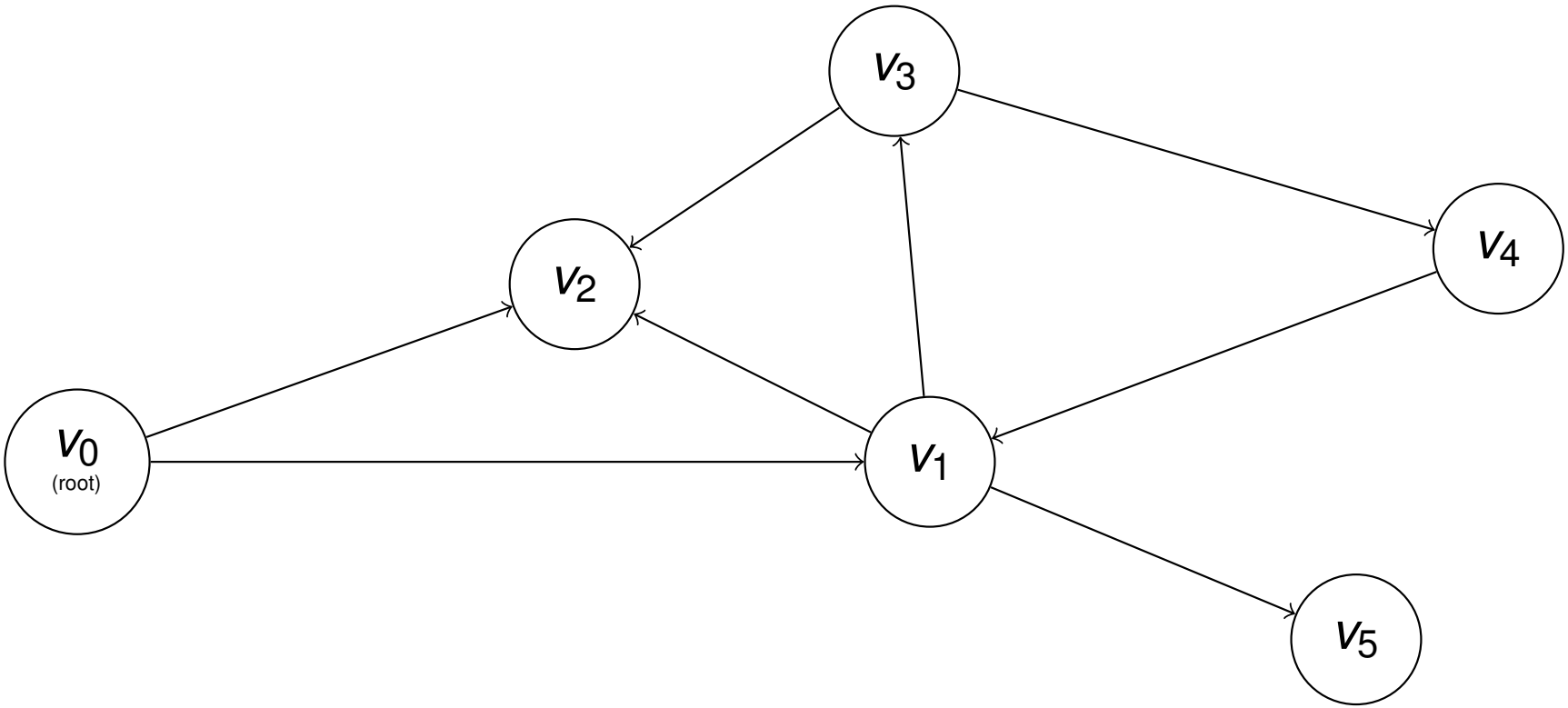
This allows us to detect cycles in directed graphs

Finding Cycles



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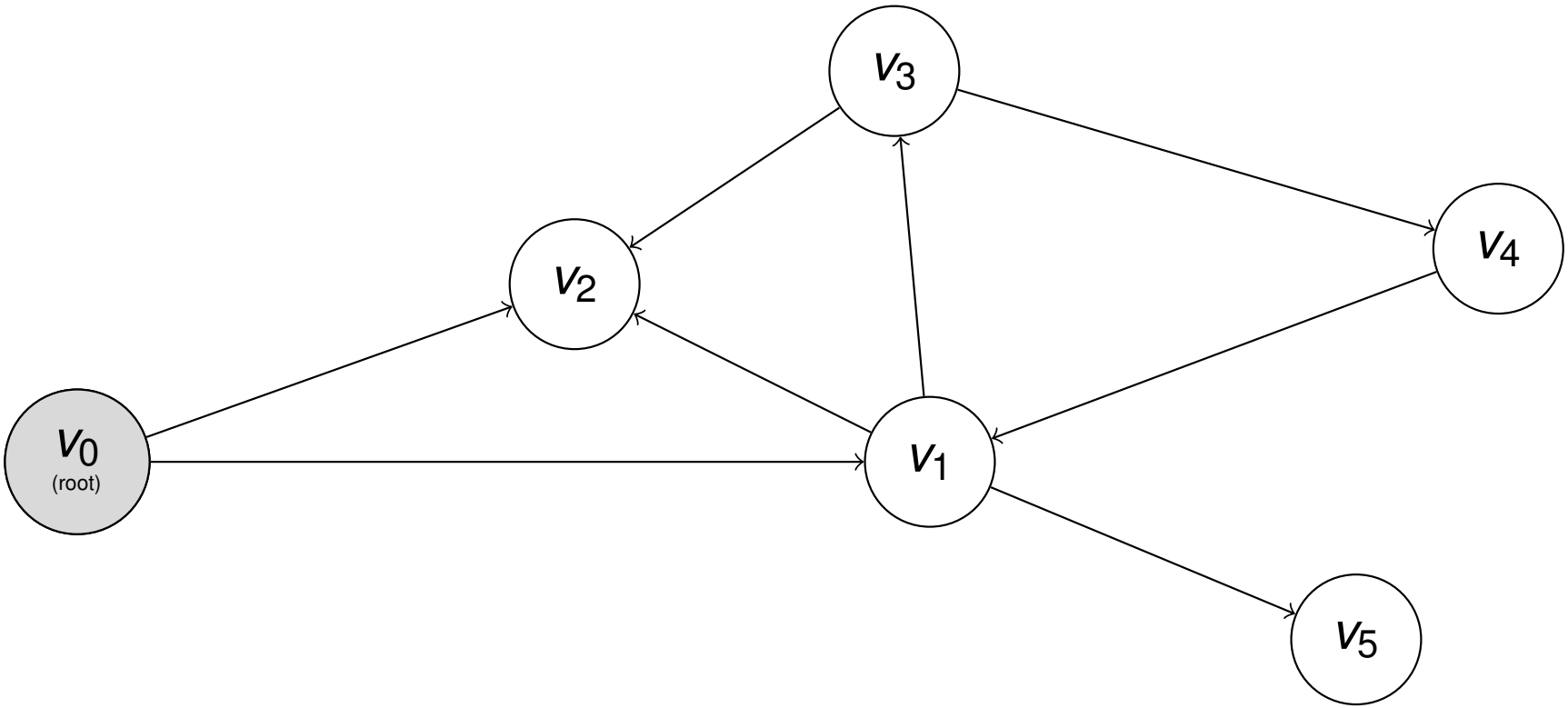

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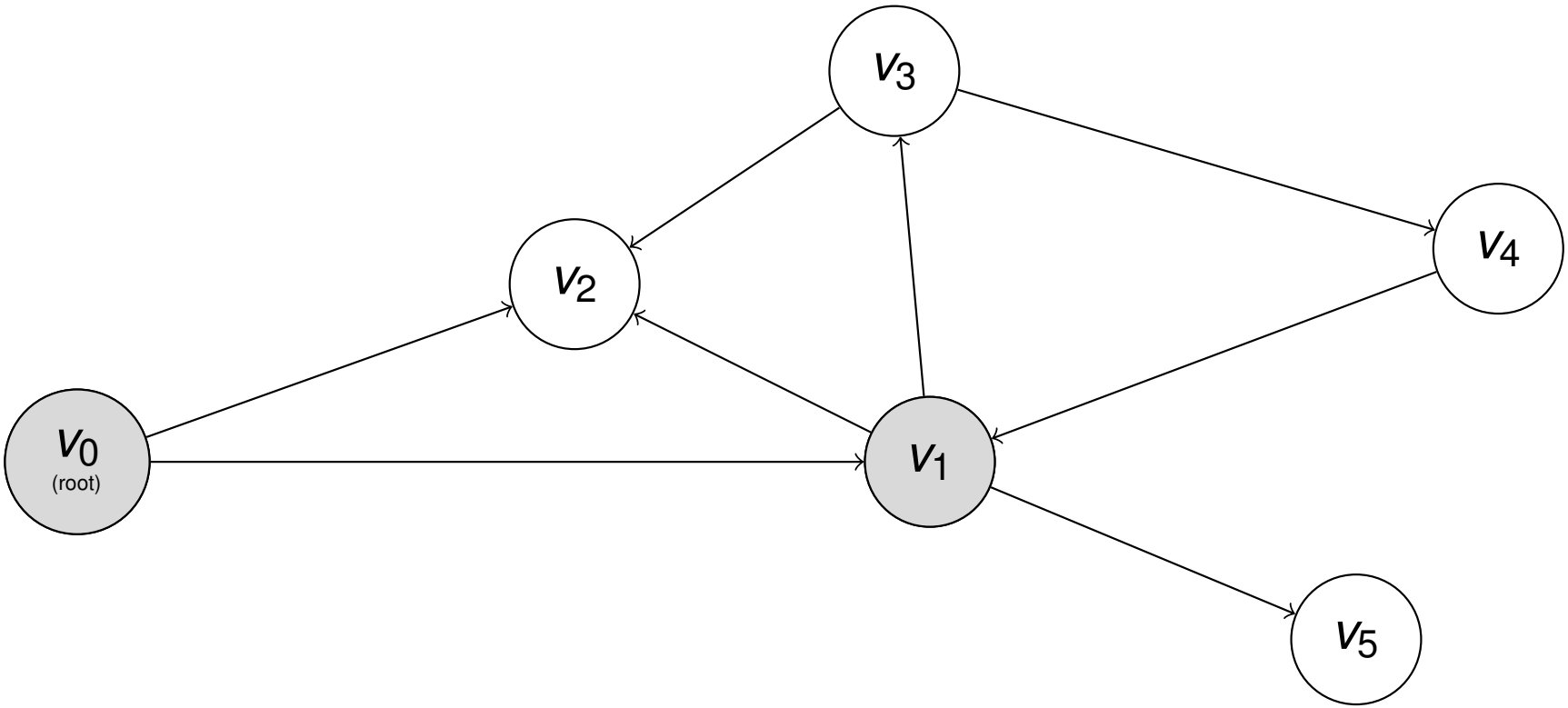

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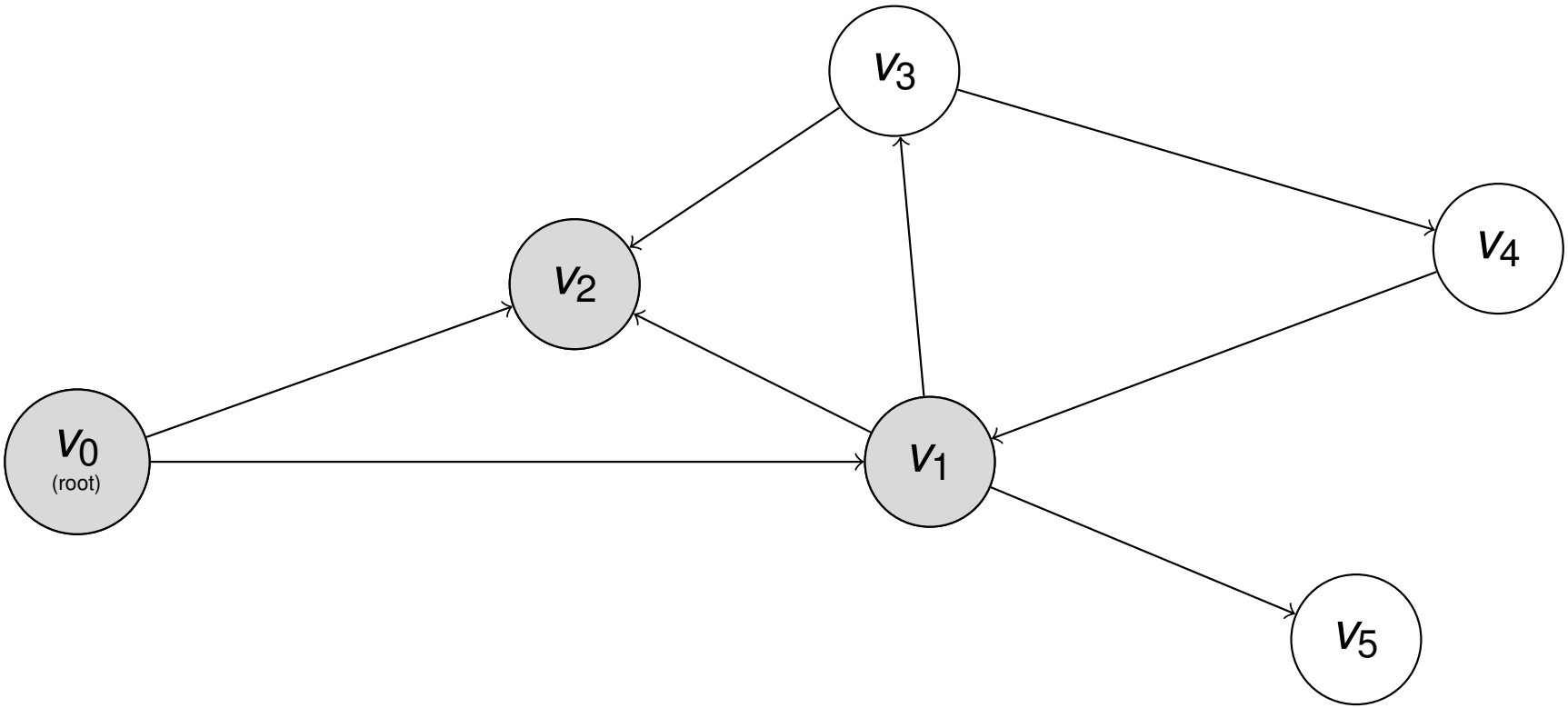

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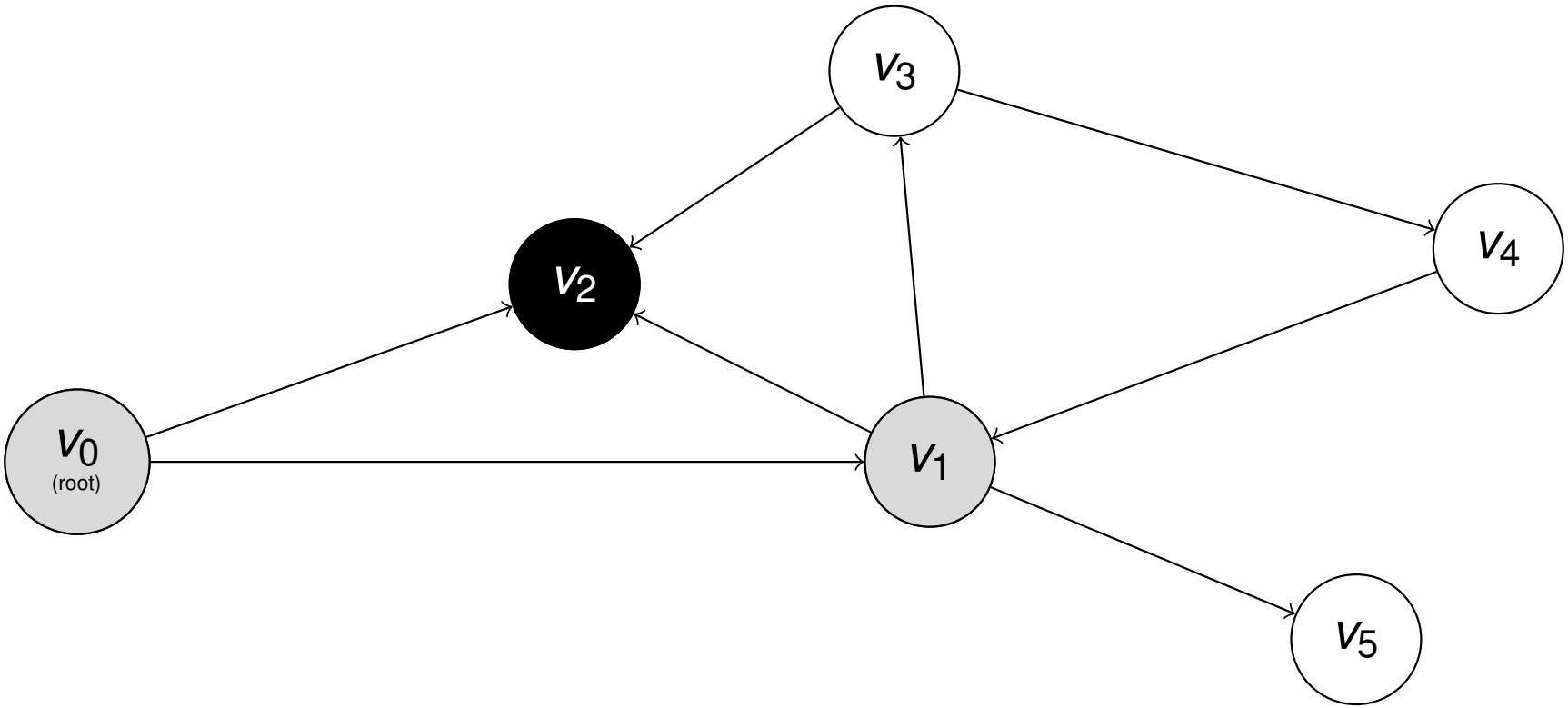

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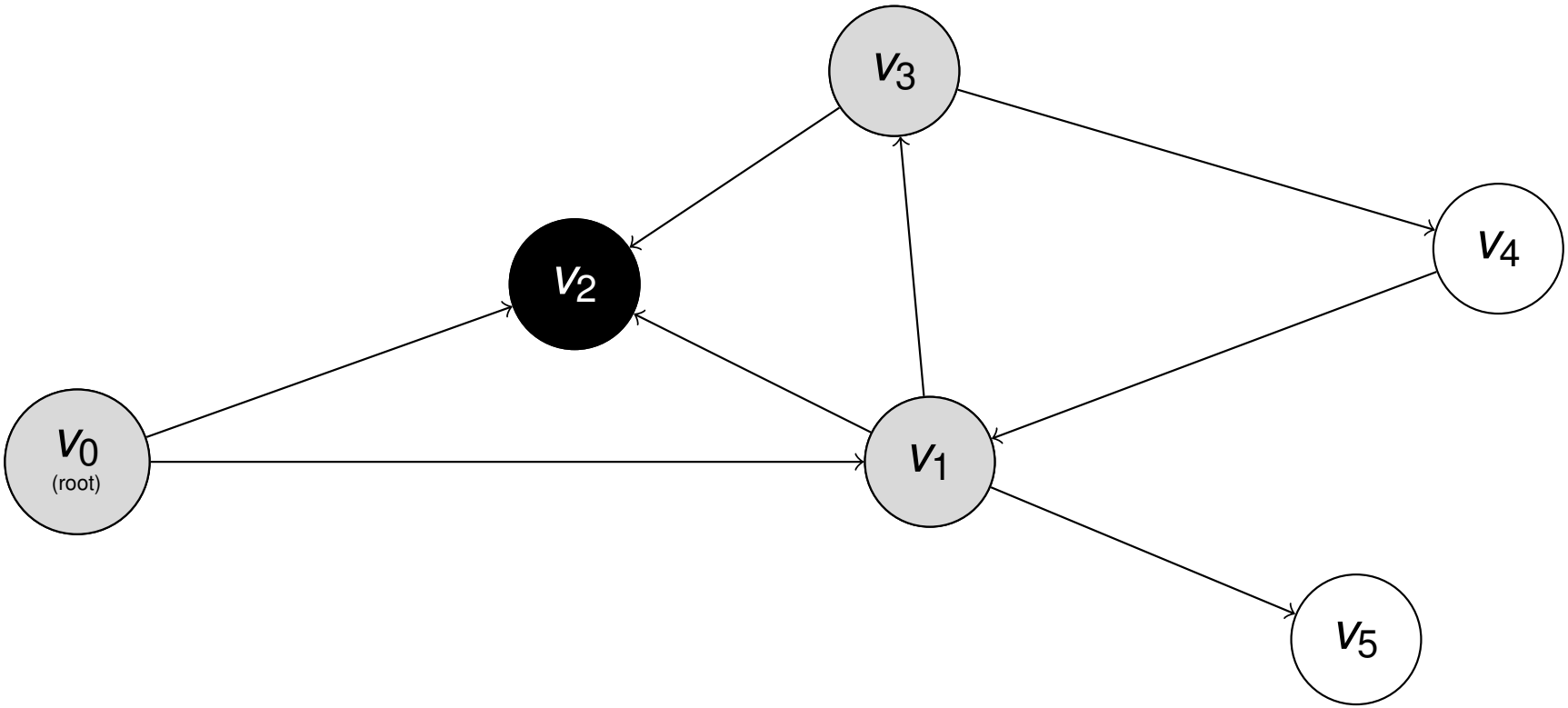

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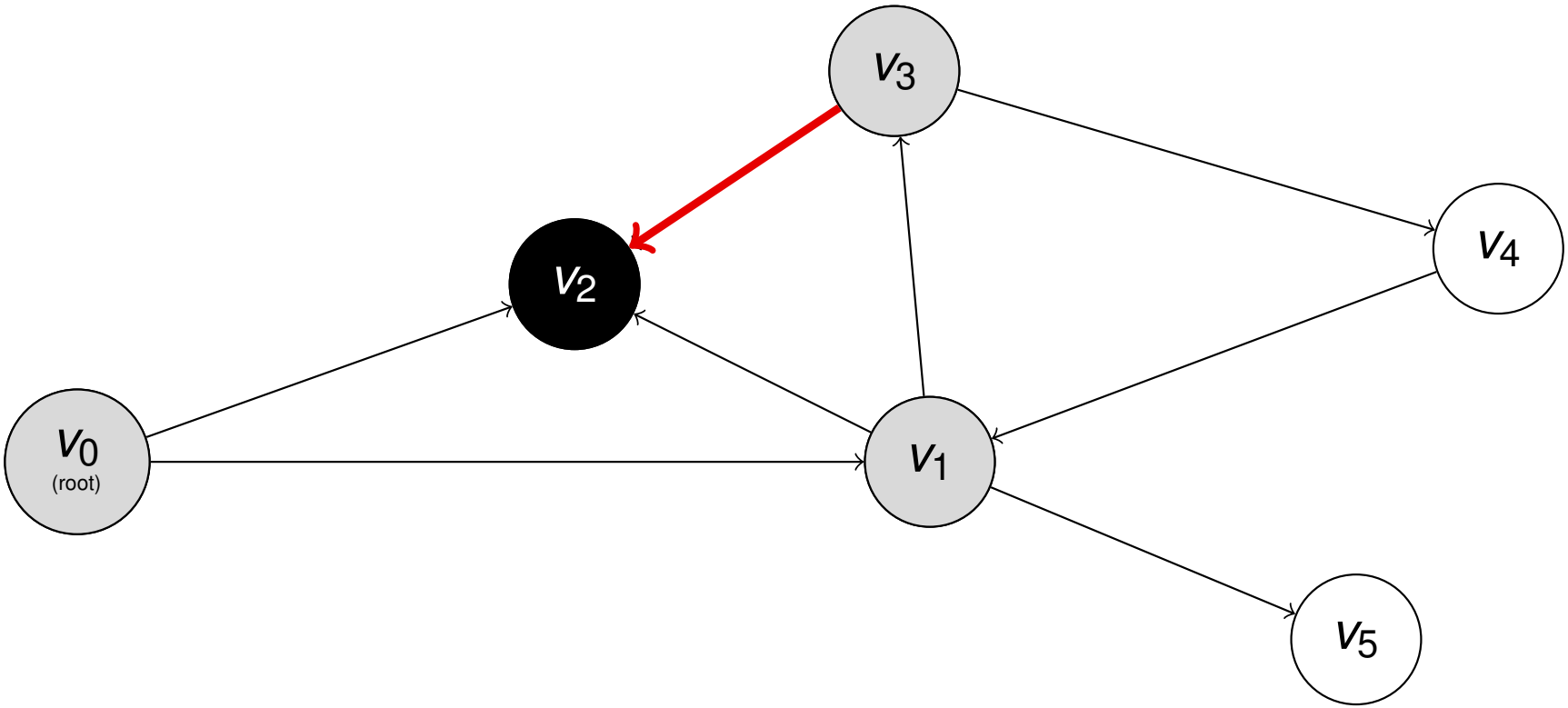

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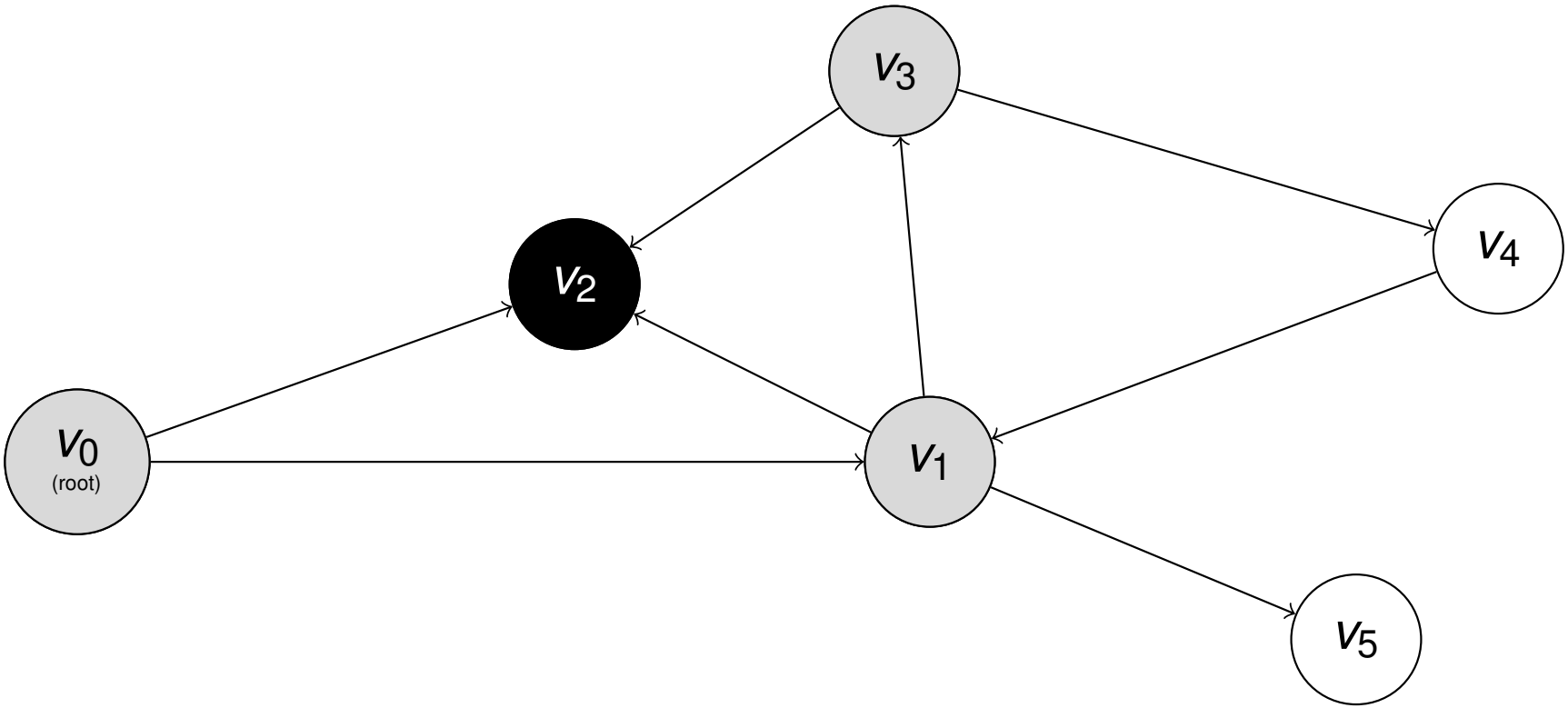

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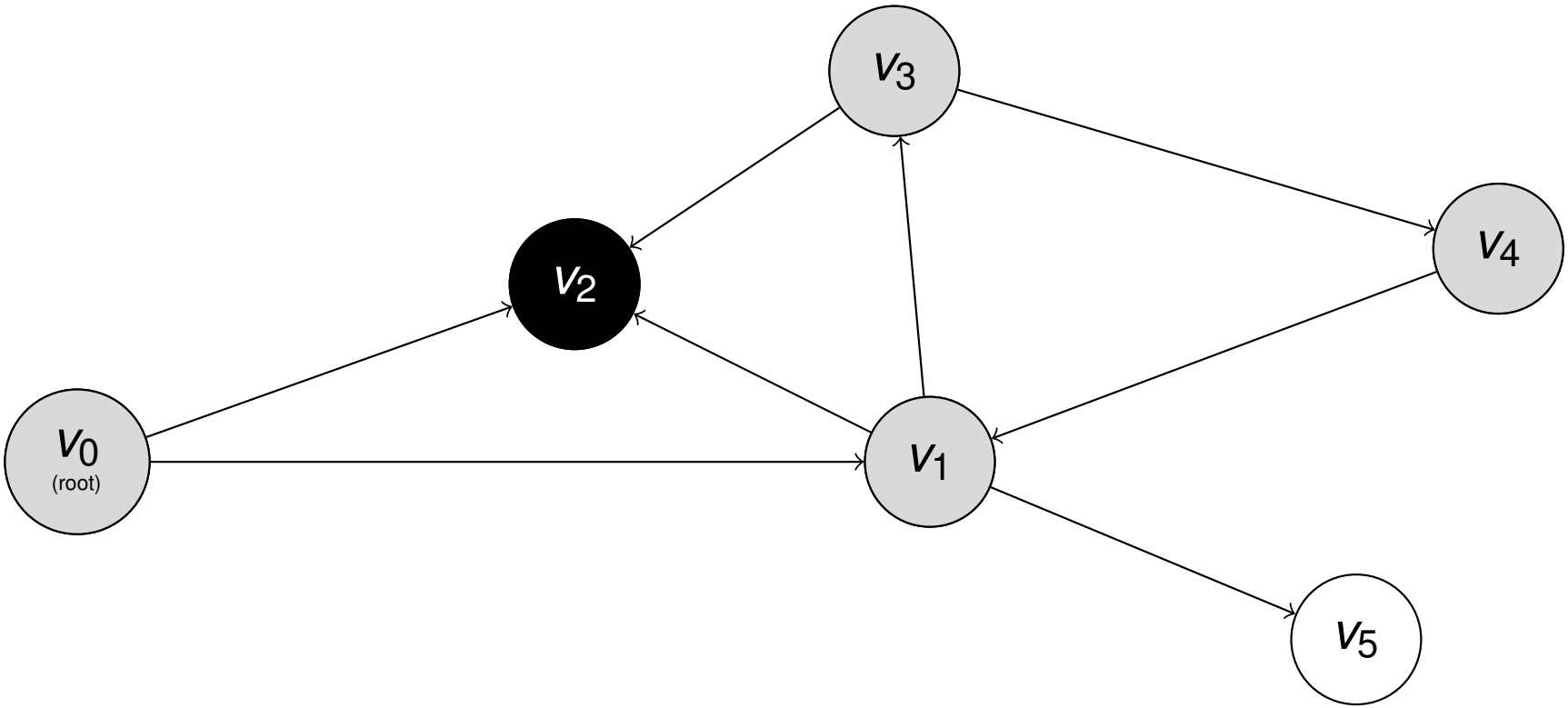

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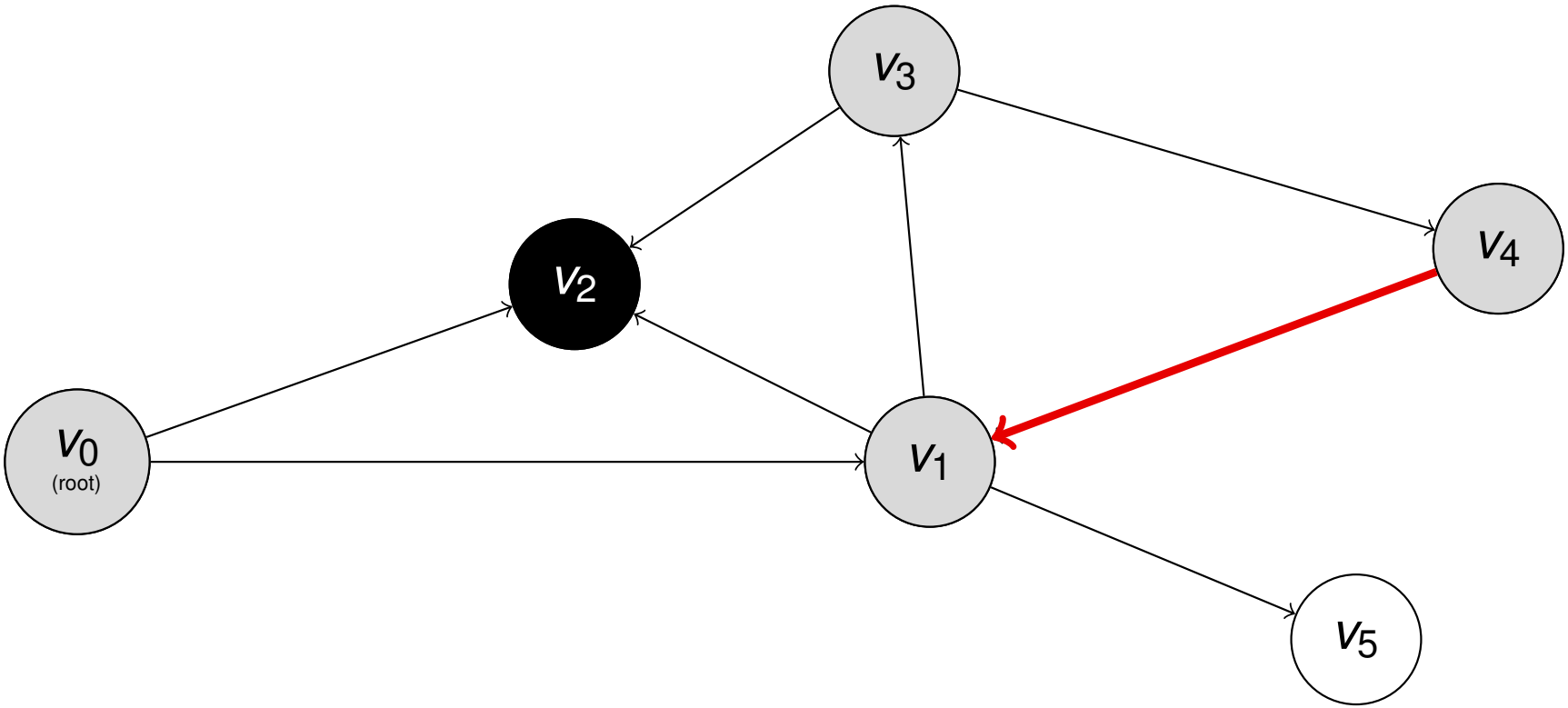

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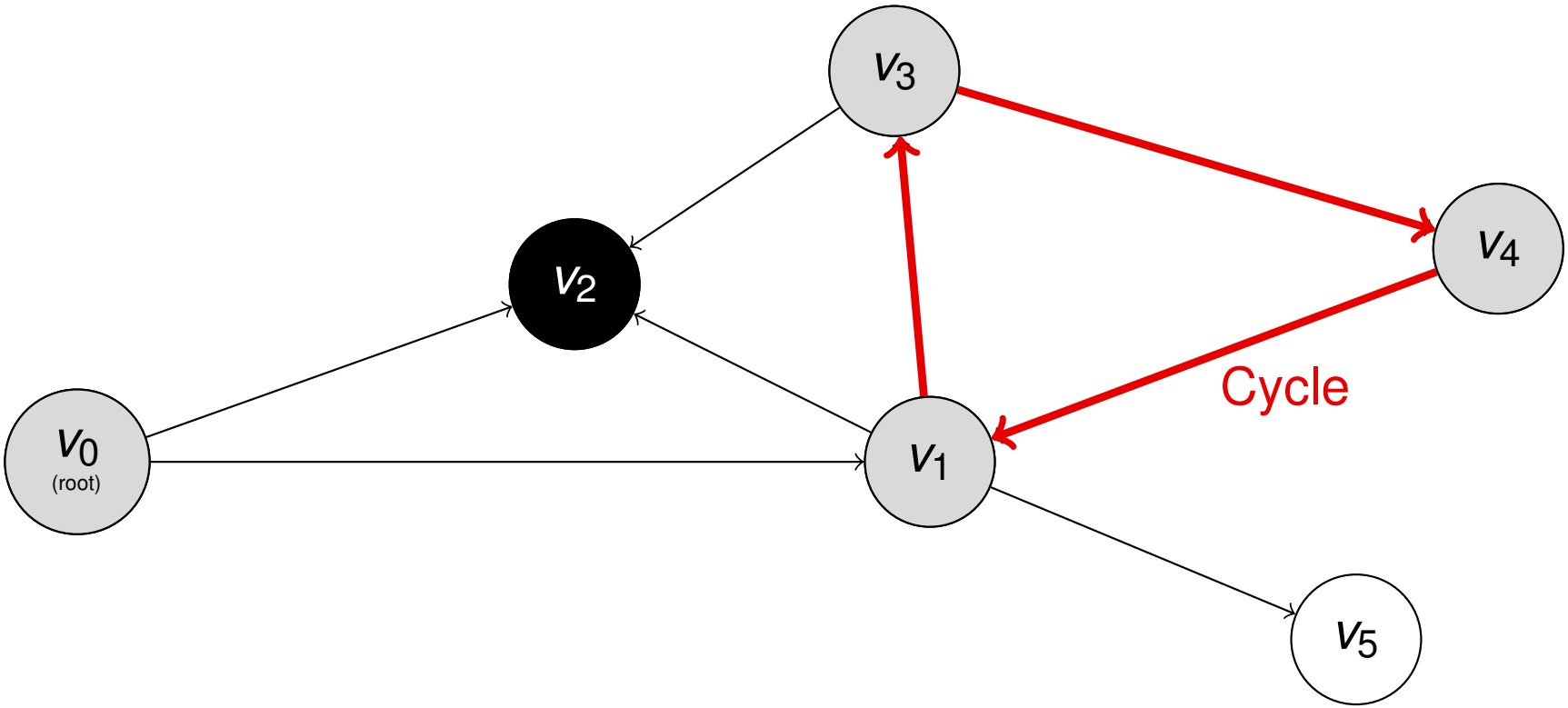

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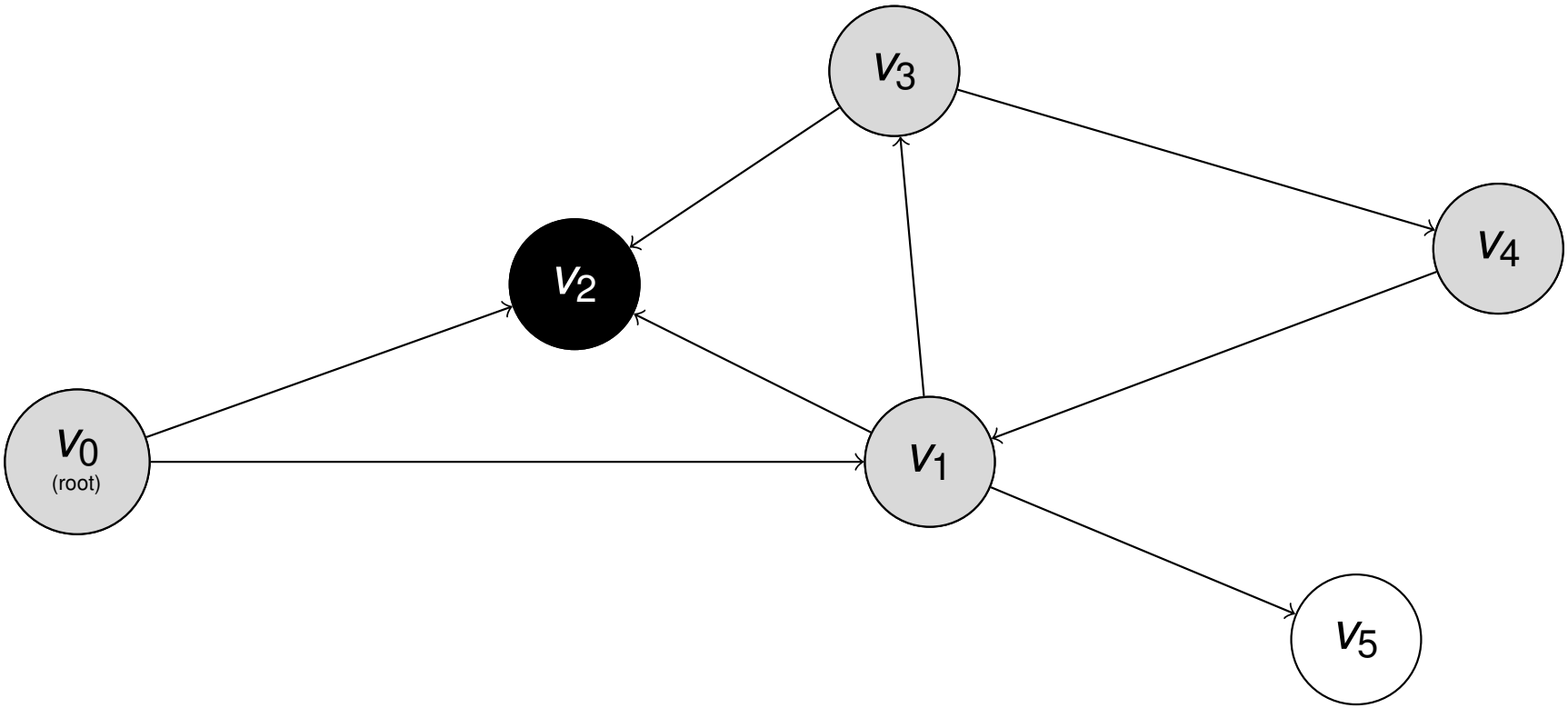

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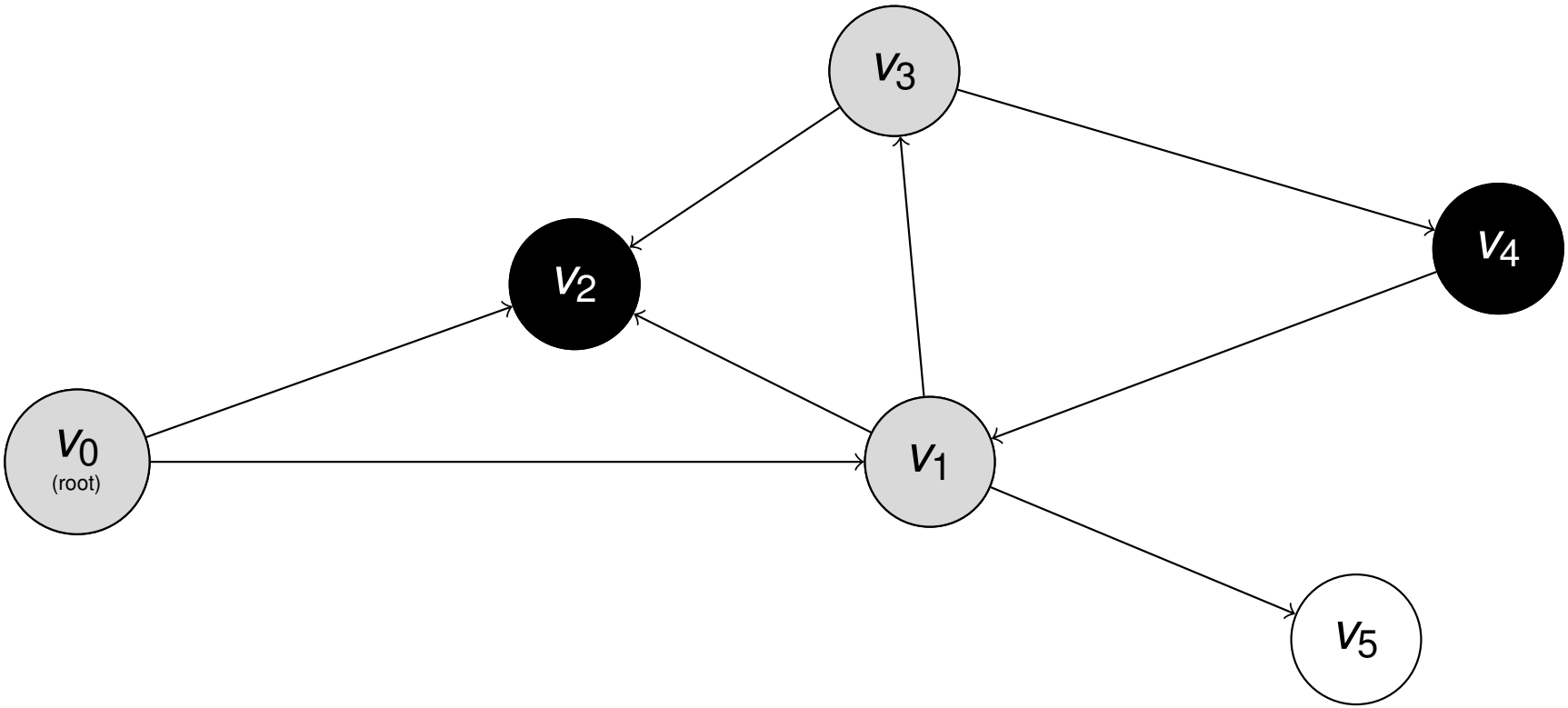

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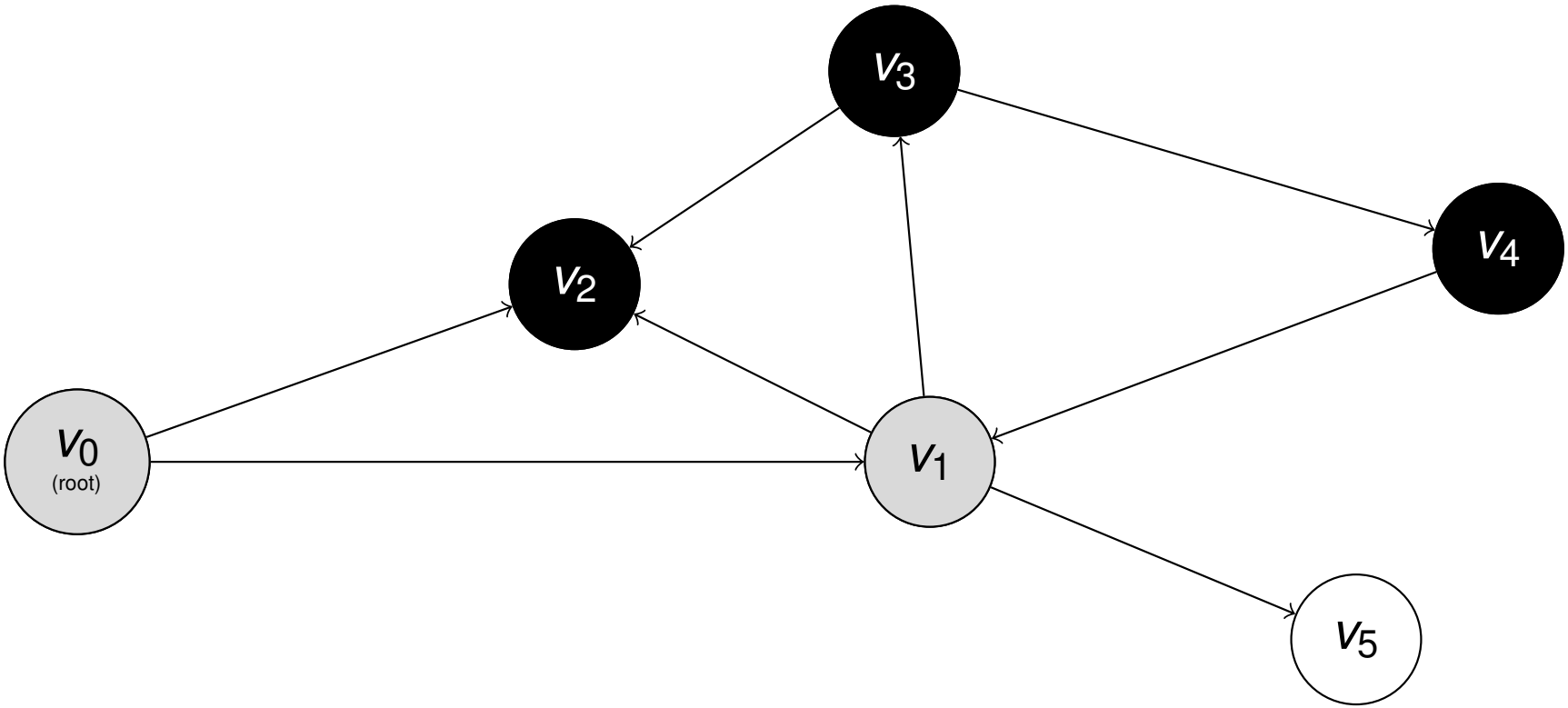

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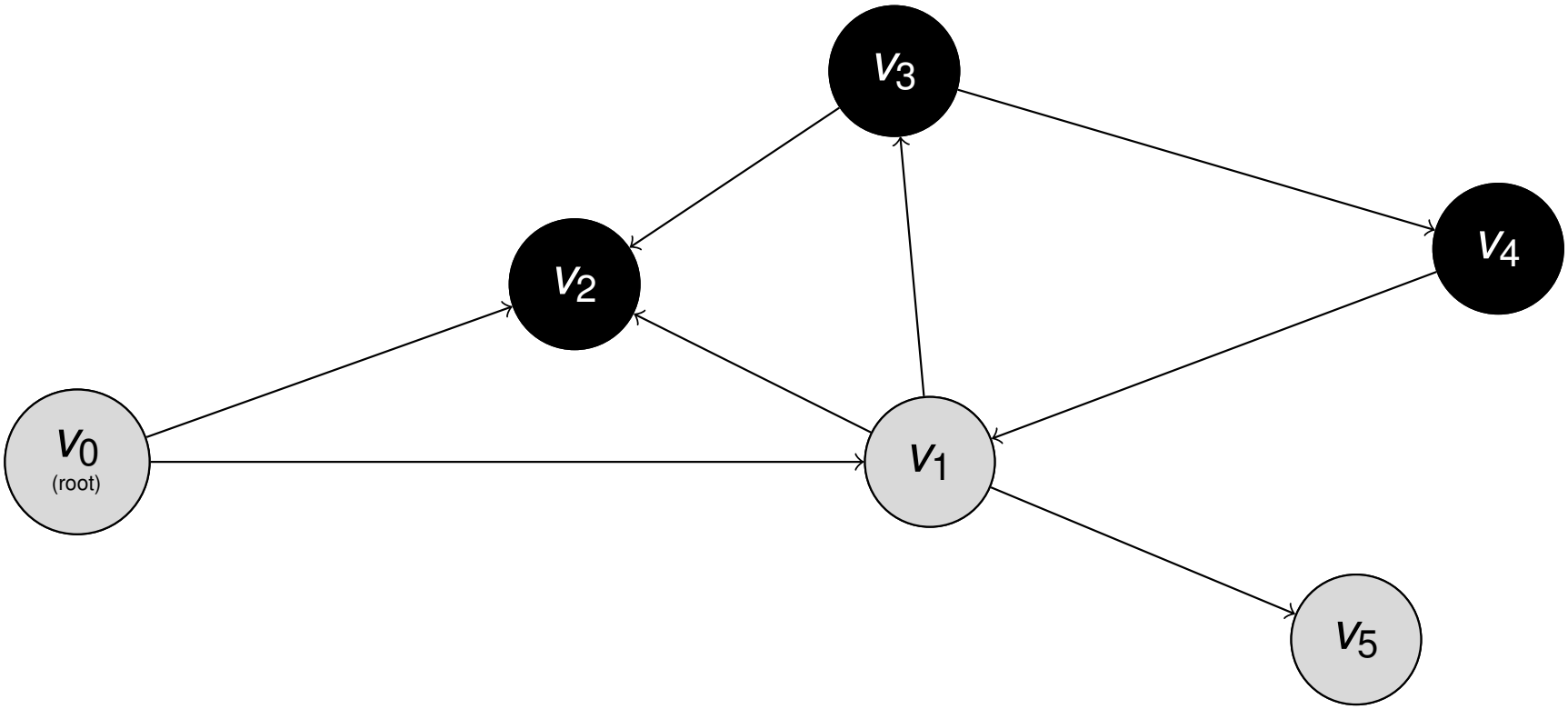

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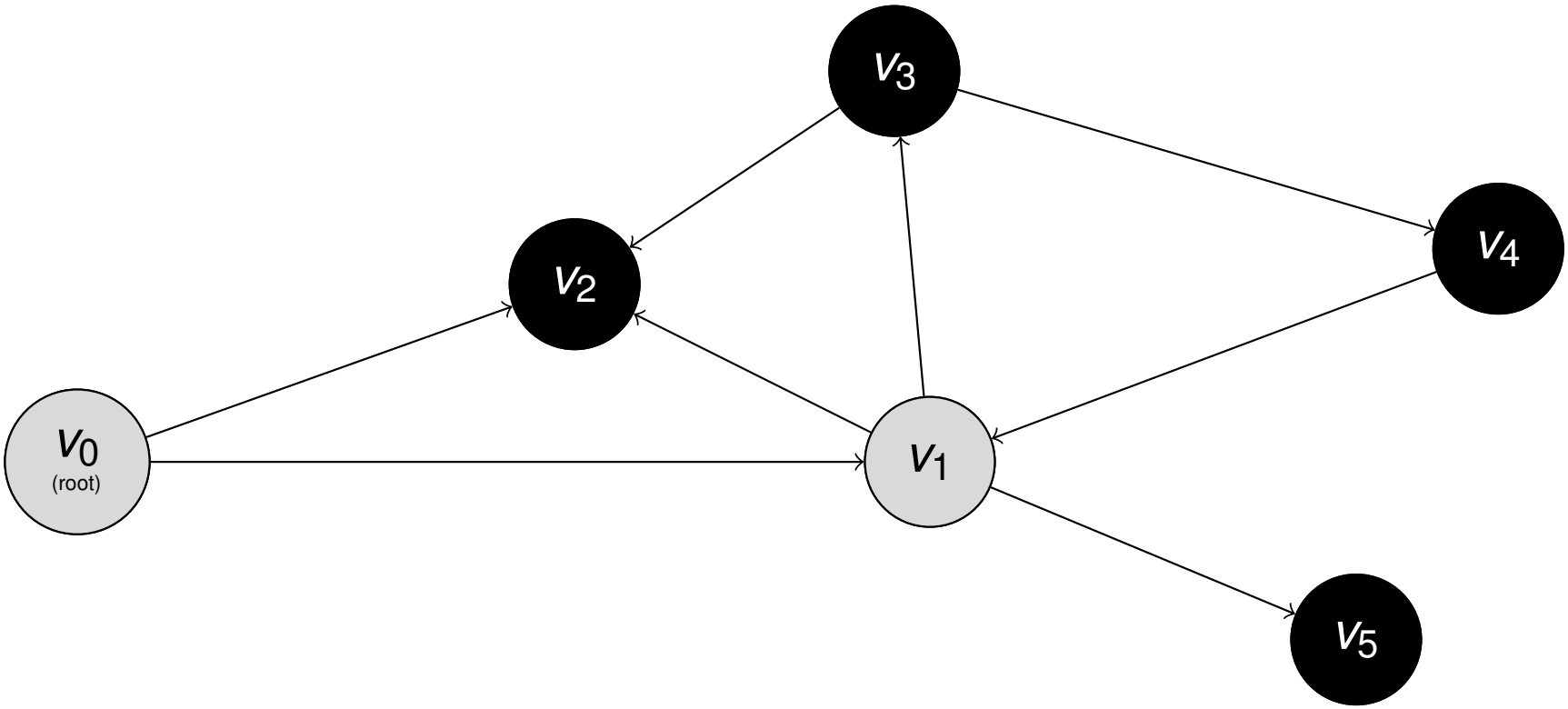

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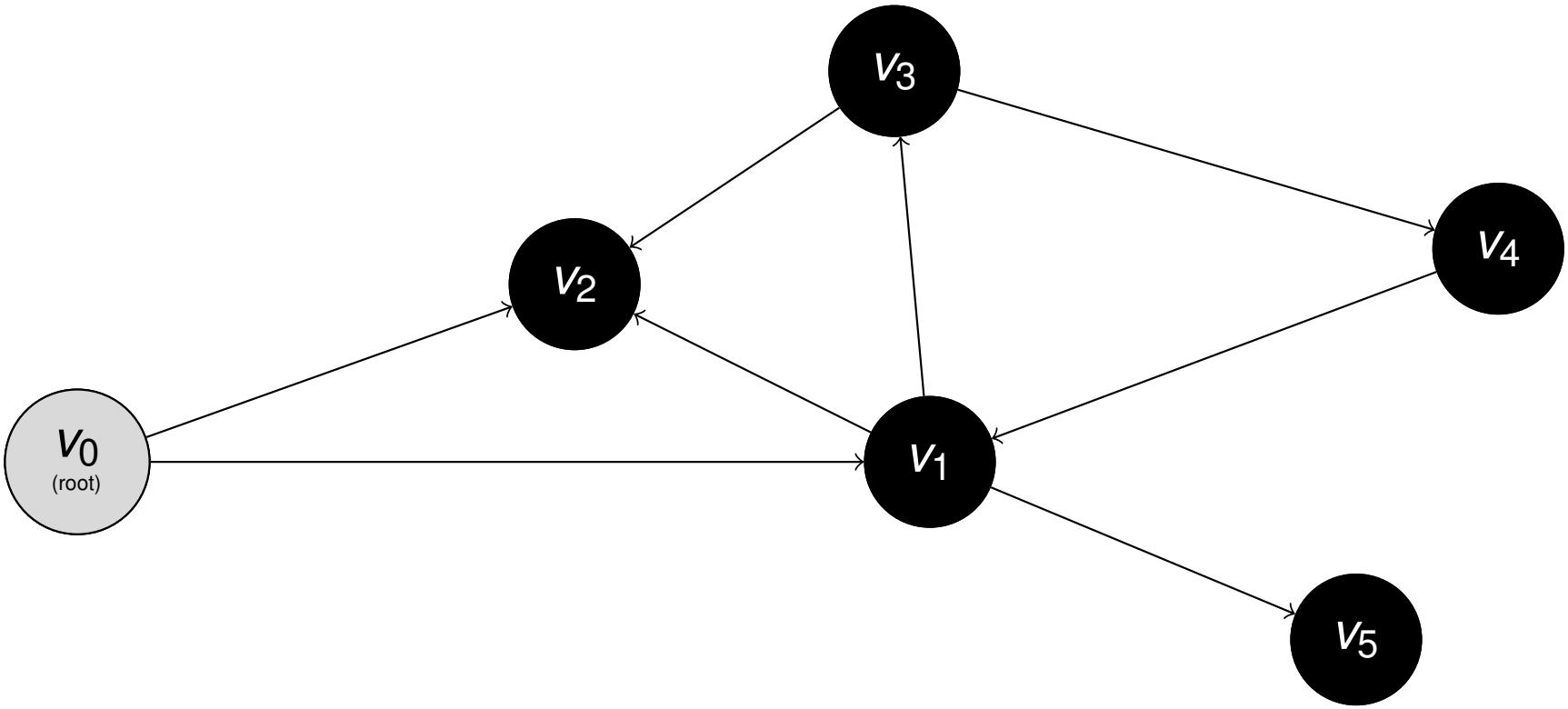

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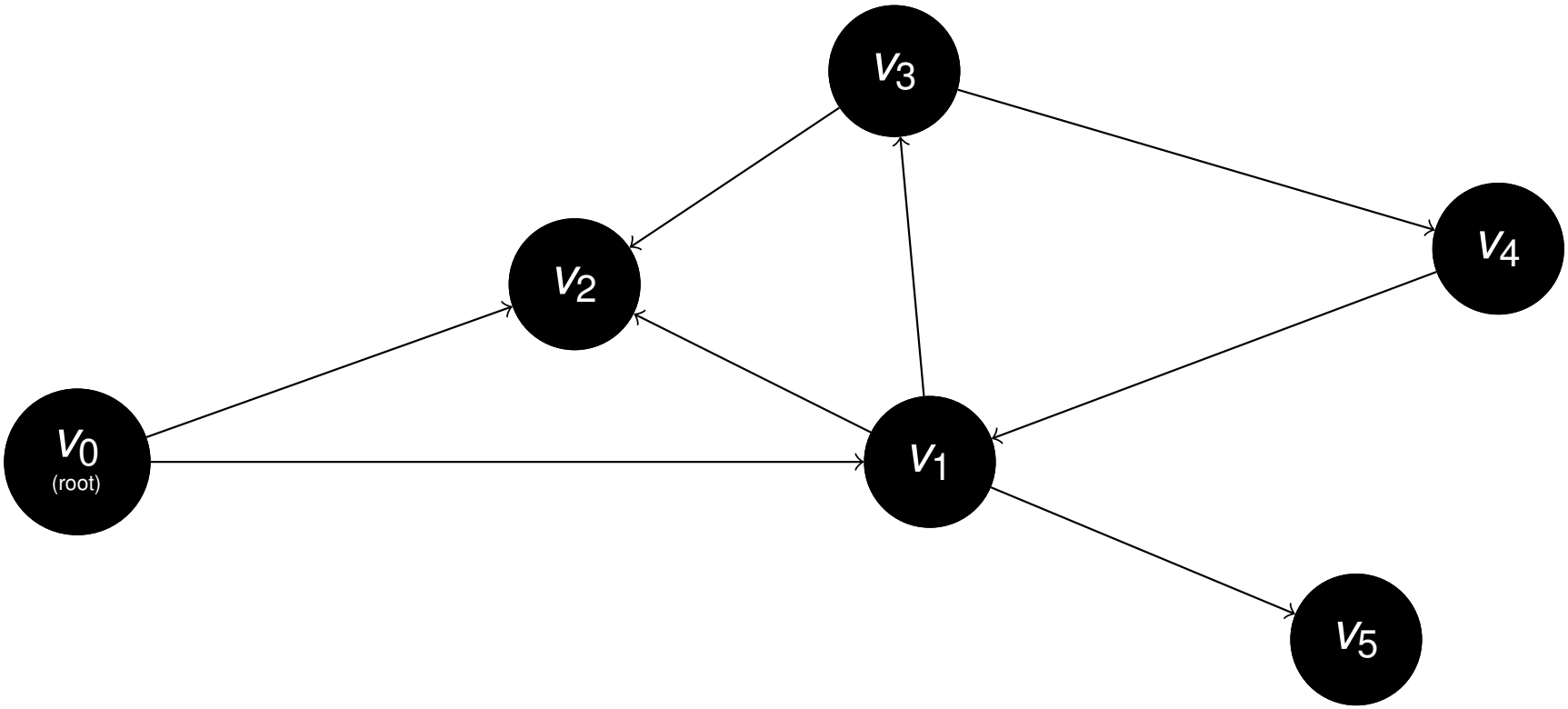

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Undirected graph has only tree and back edges

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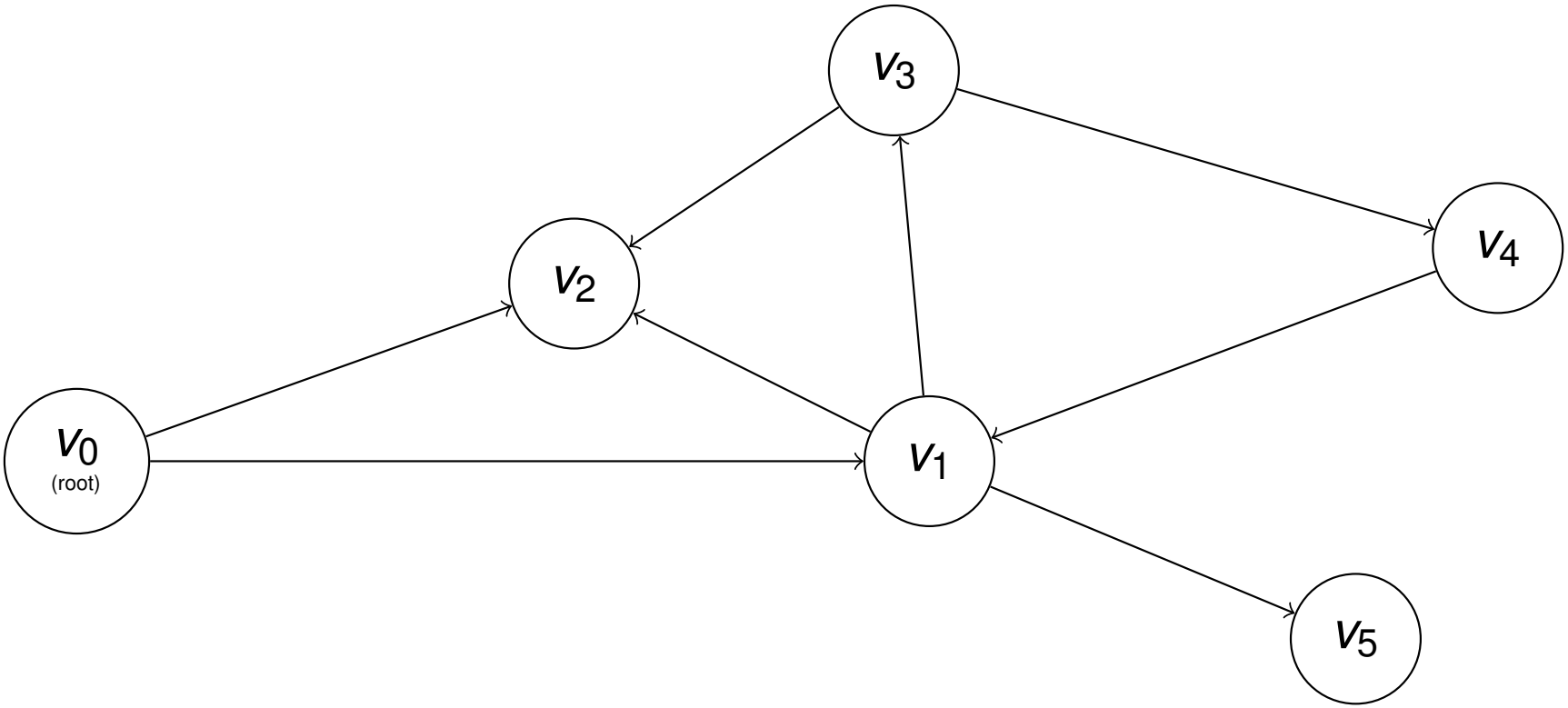

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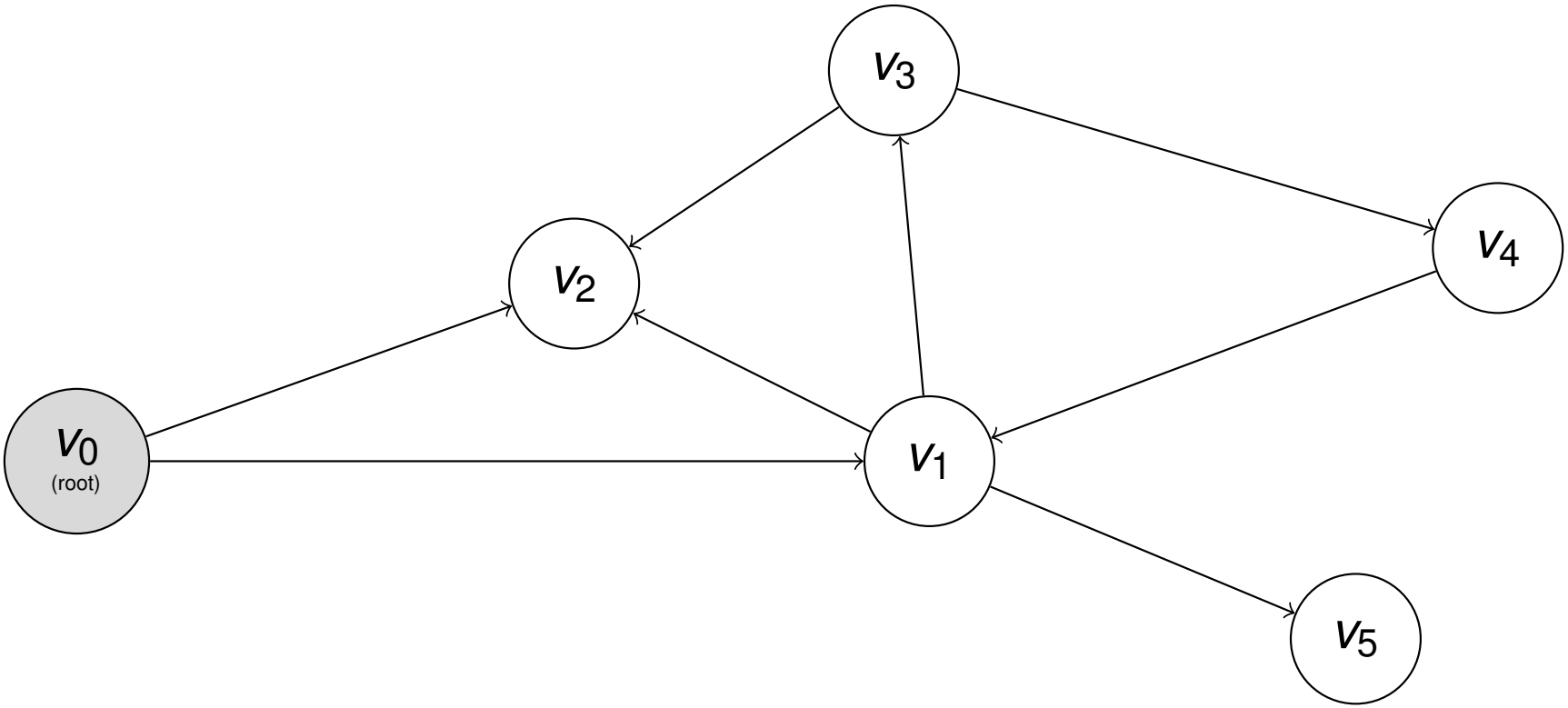

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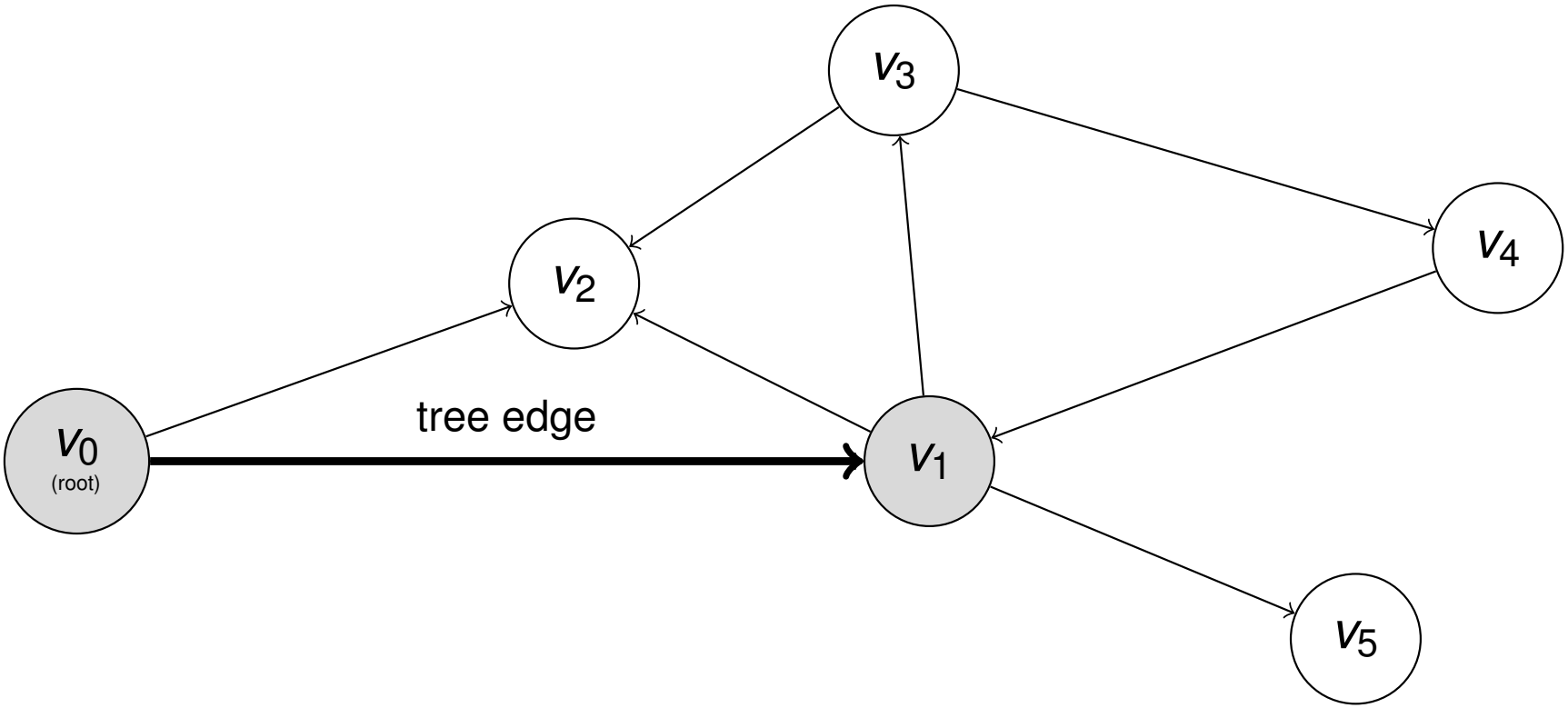

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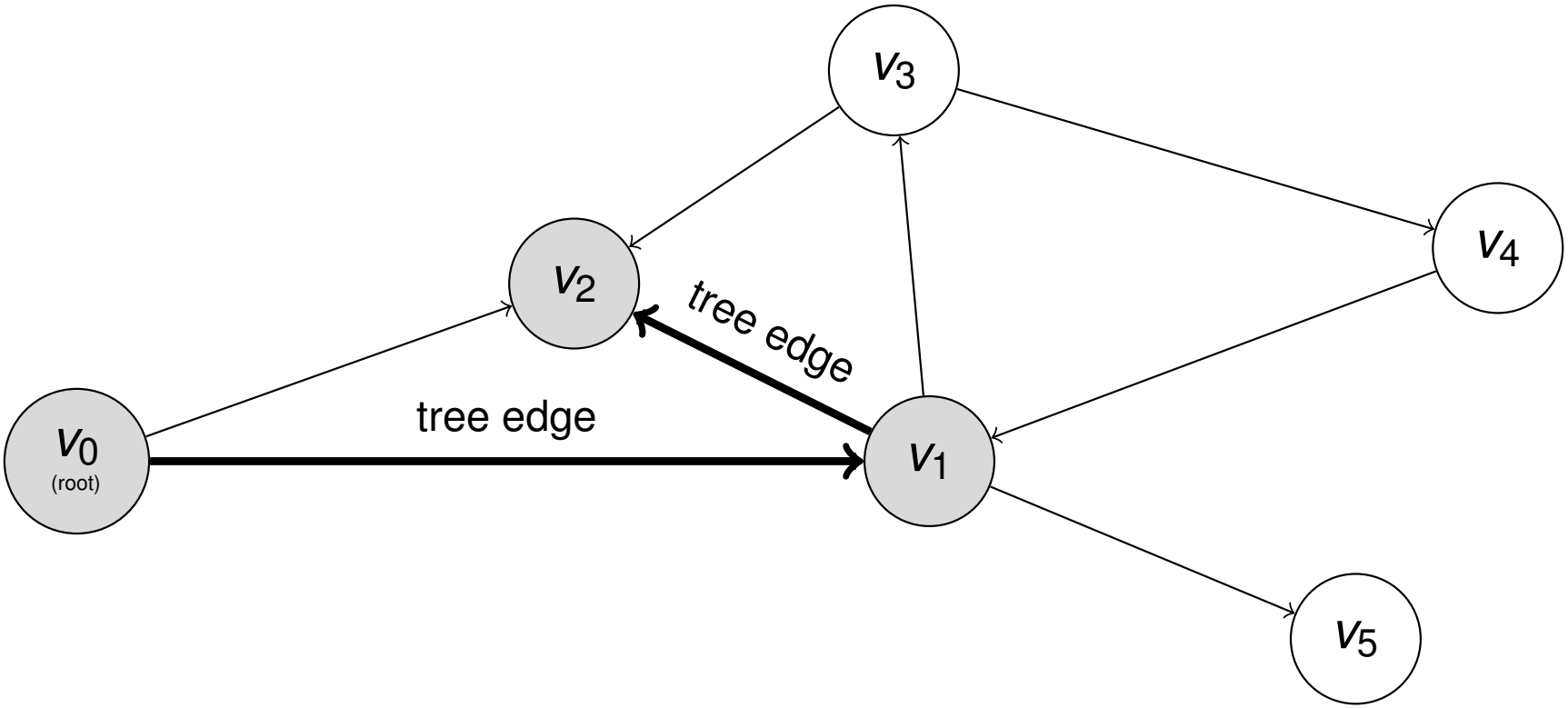

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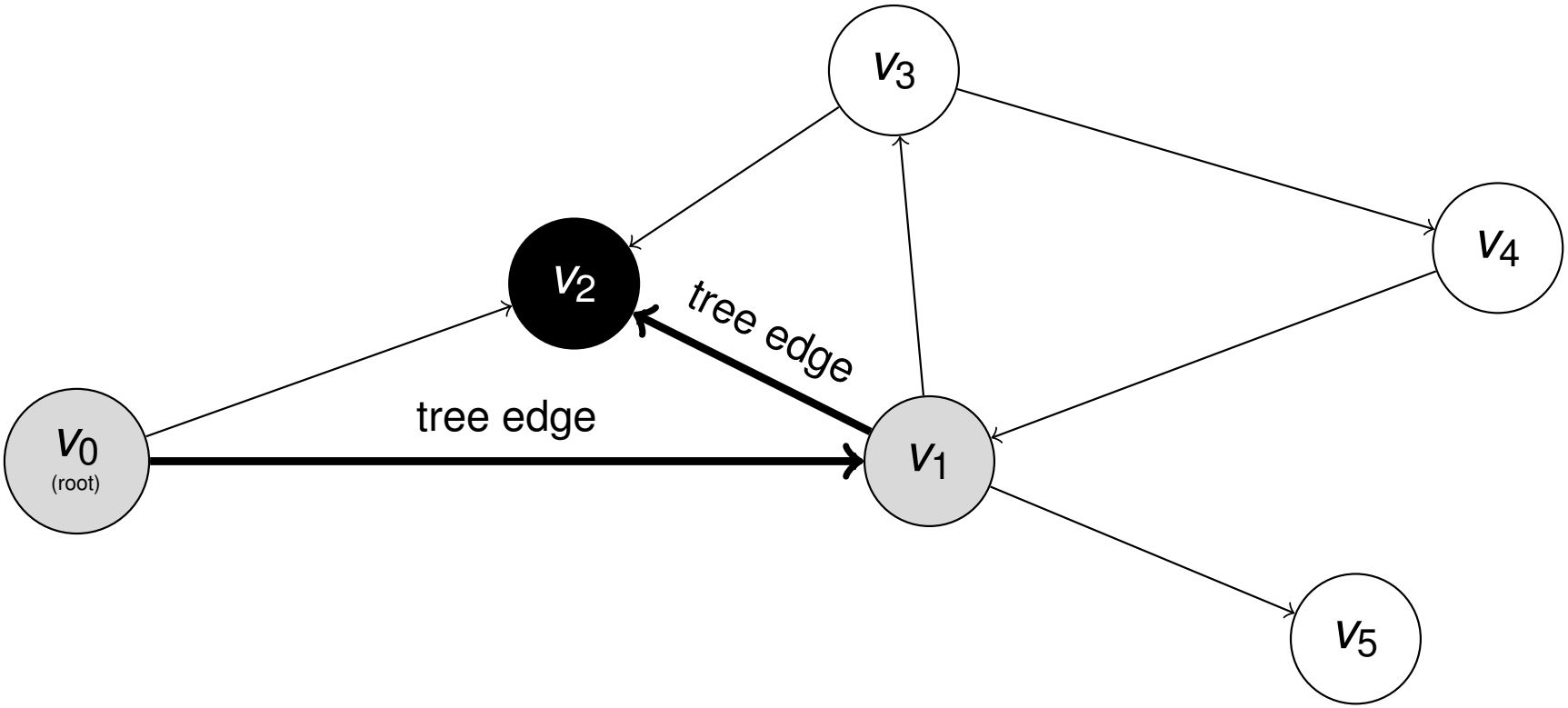

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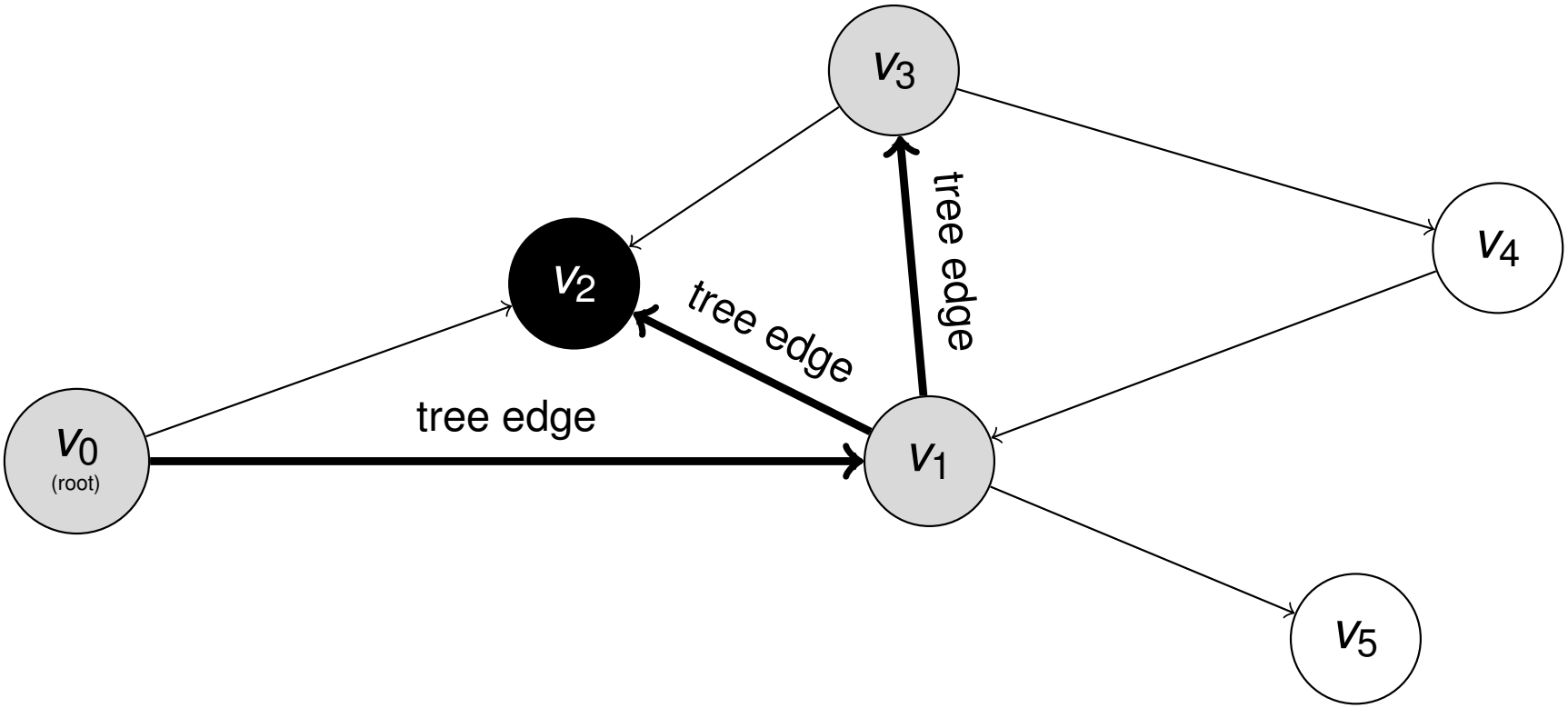

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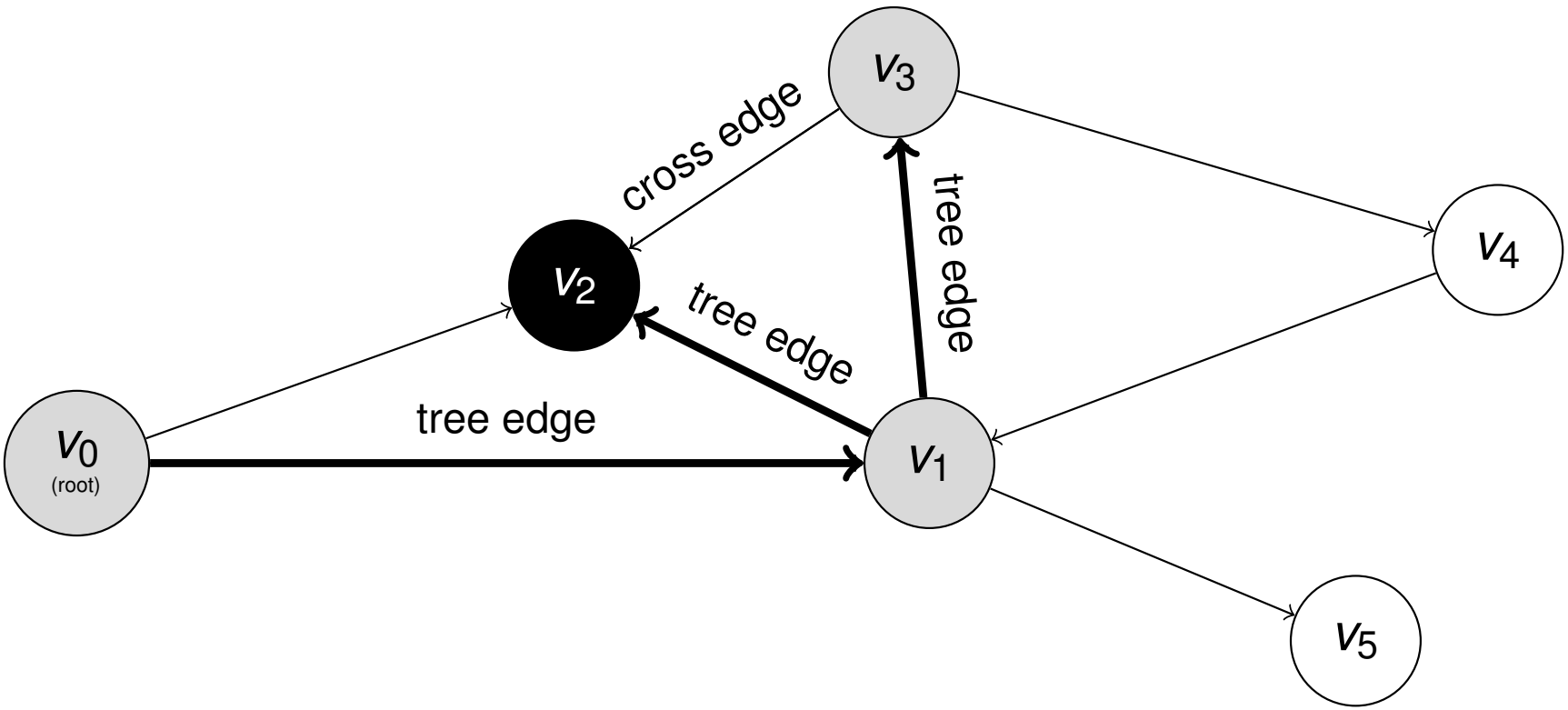

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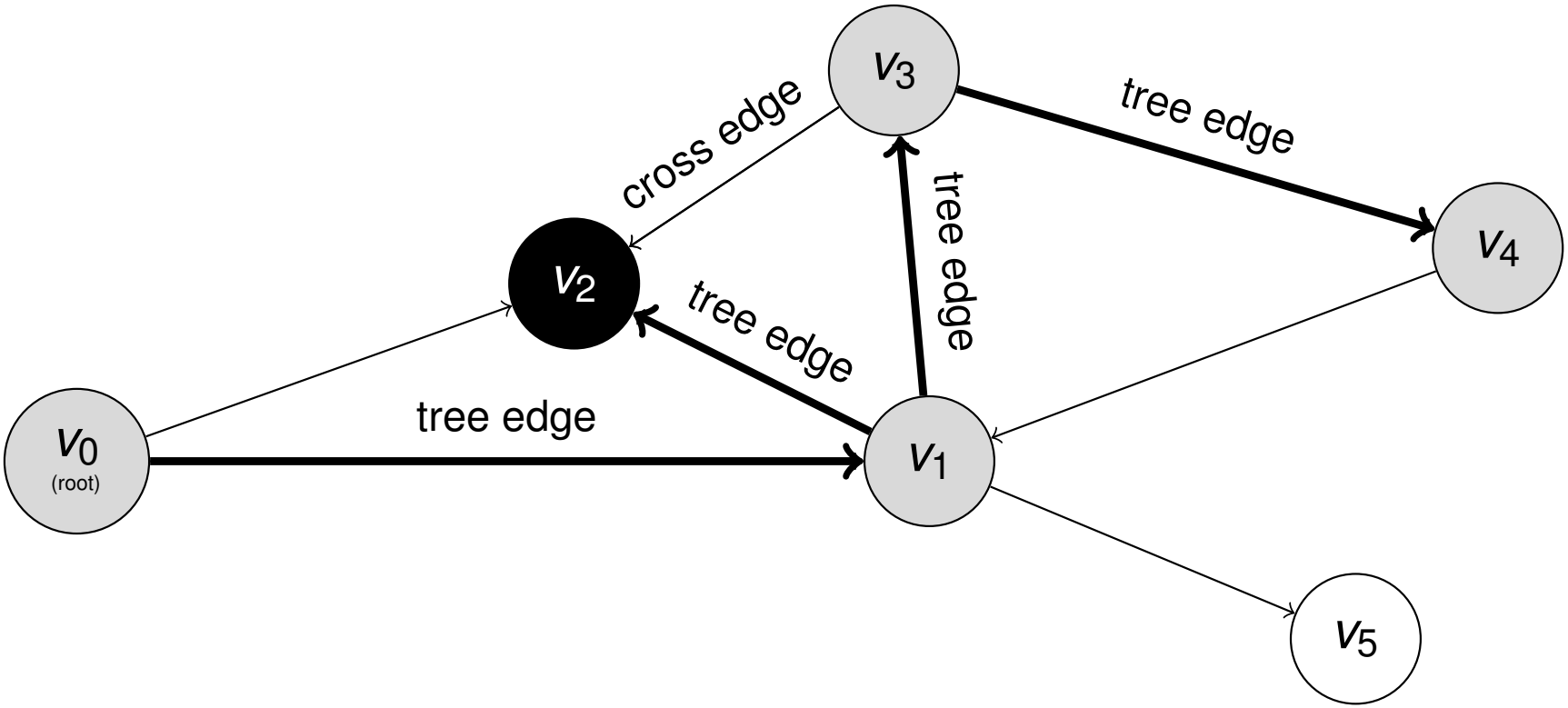

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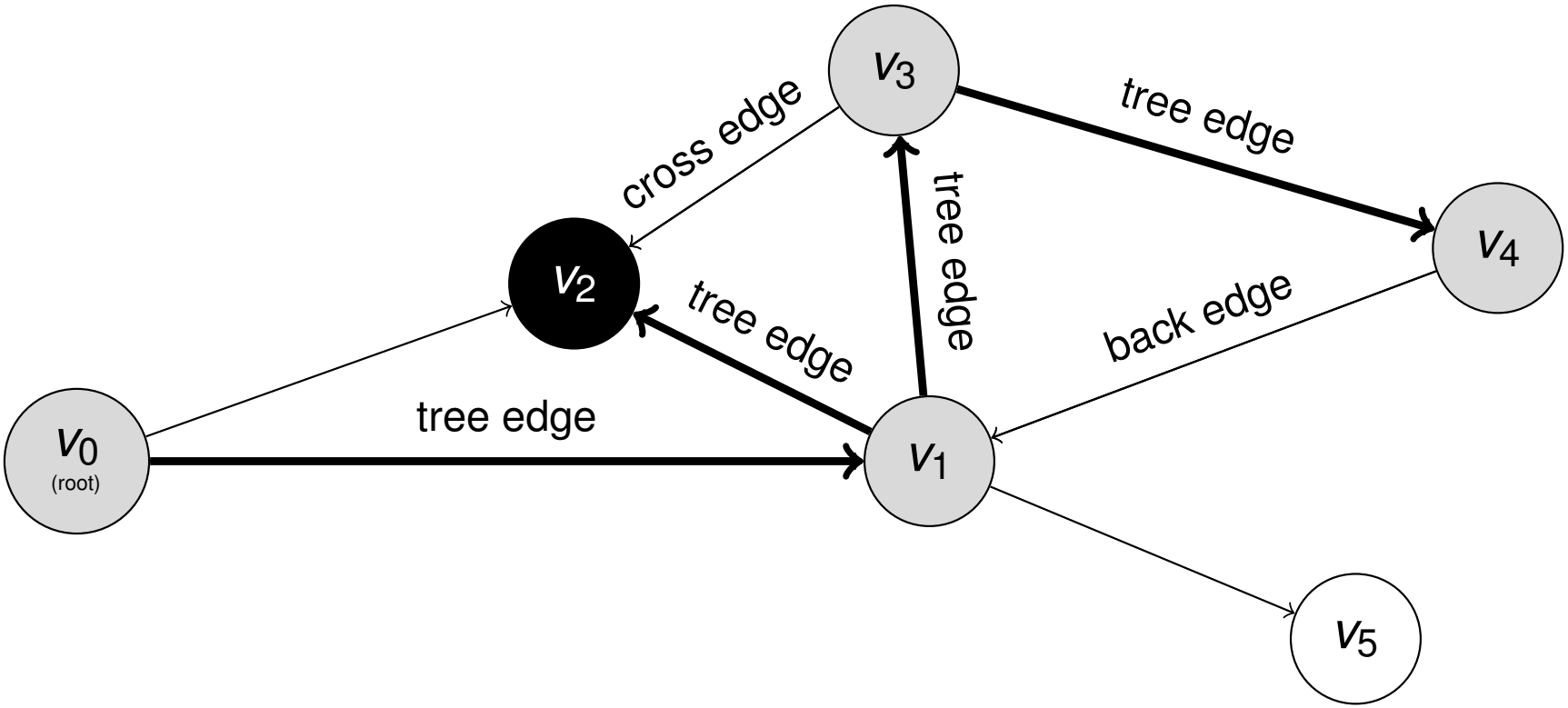

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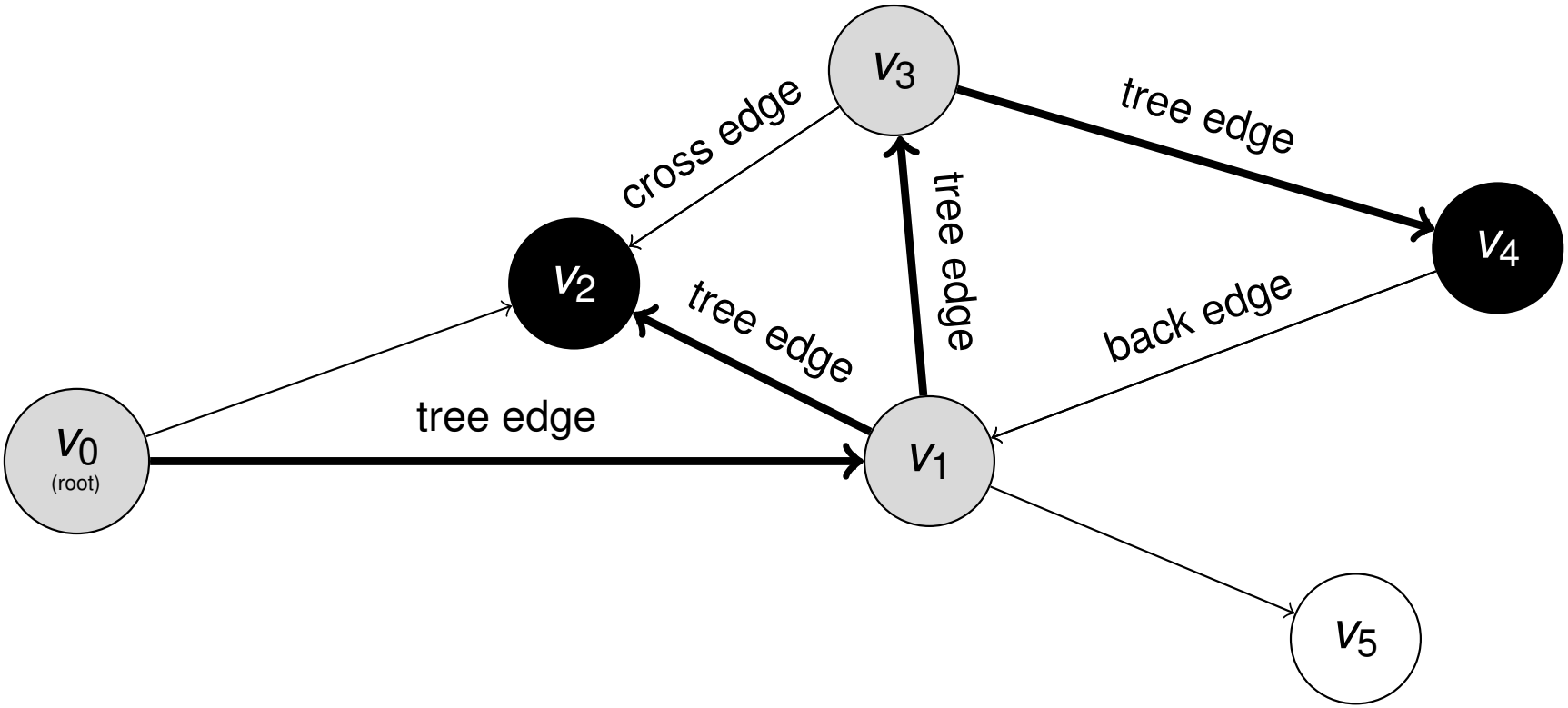

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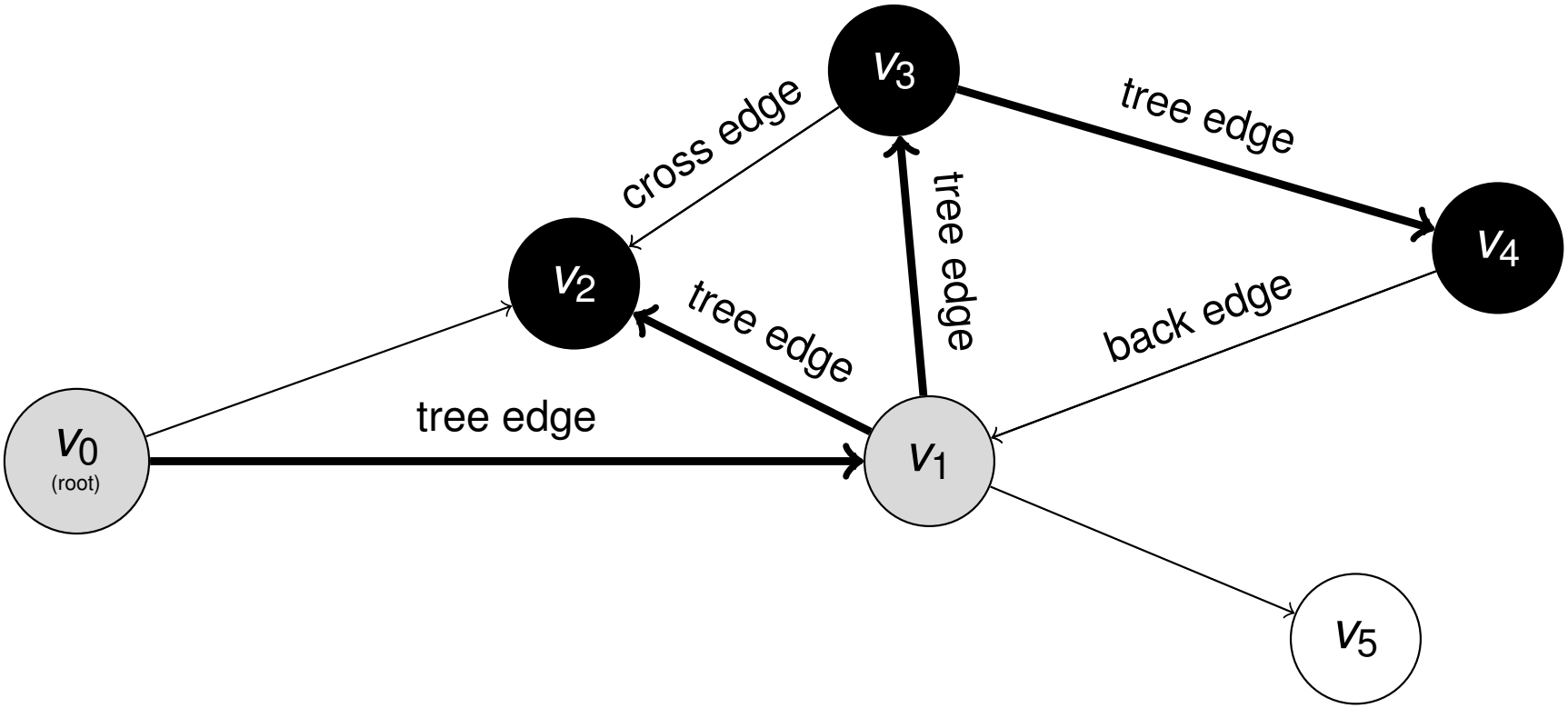

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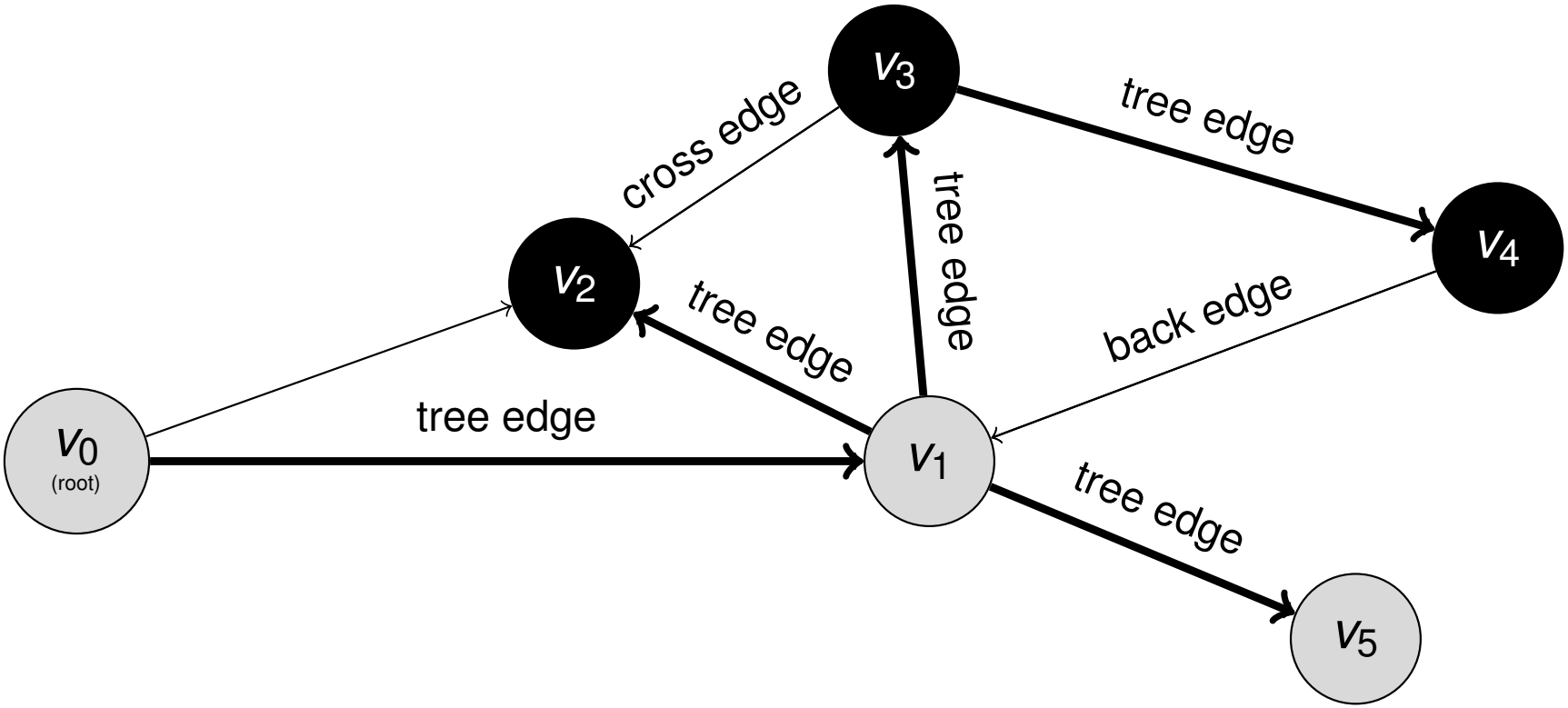

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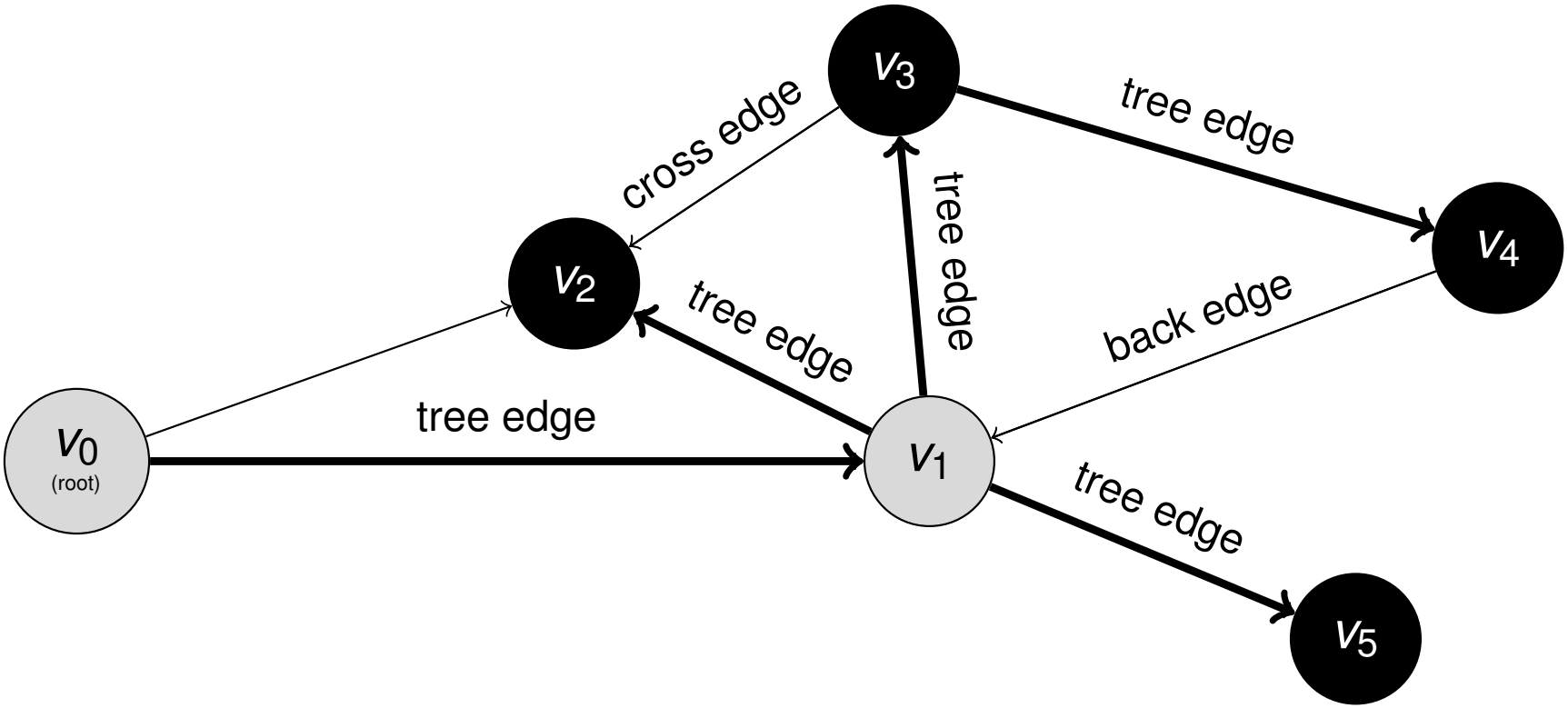

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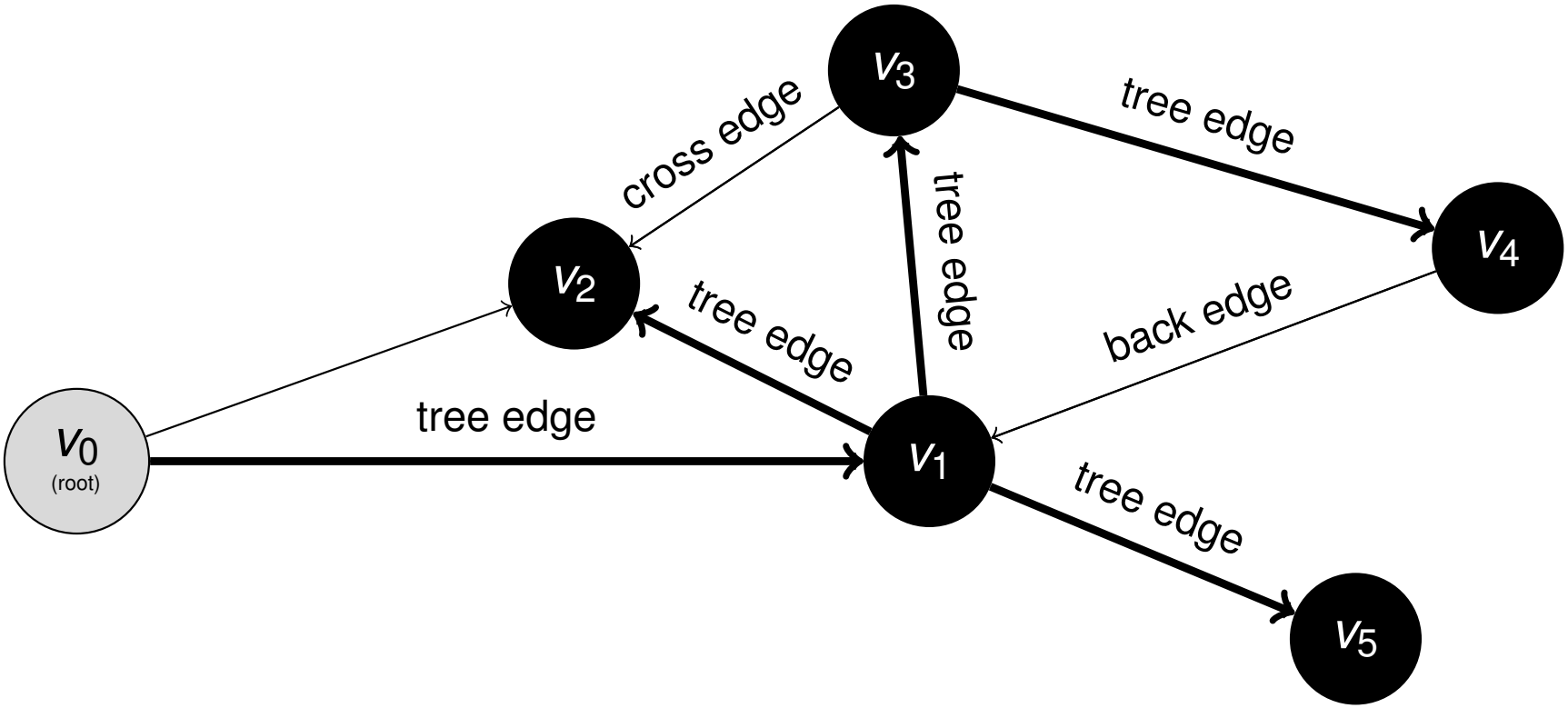

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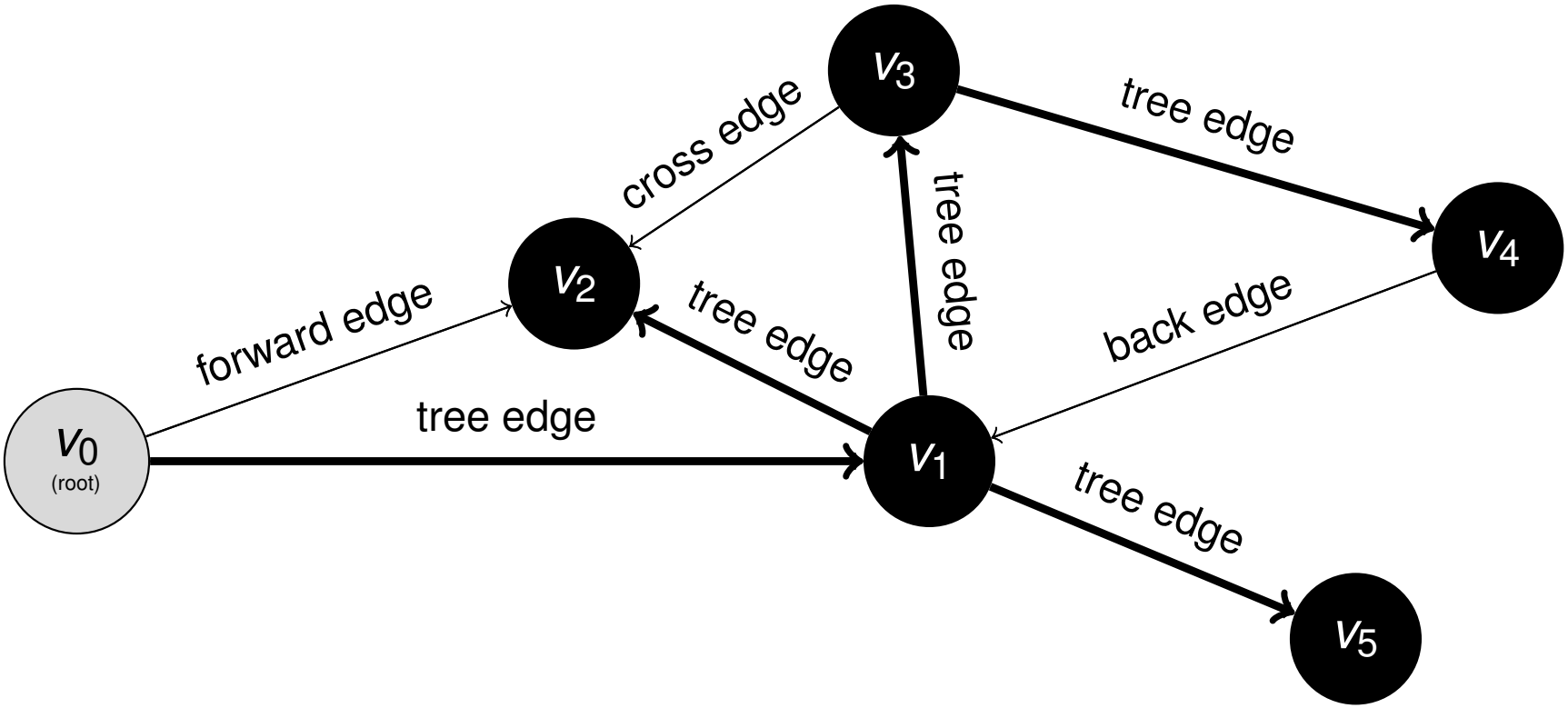

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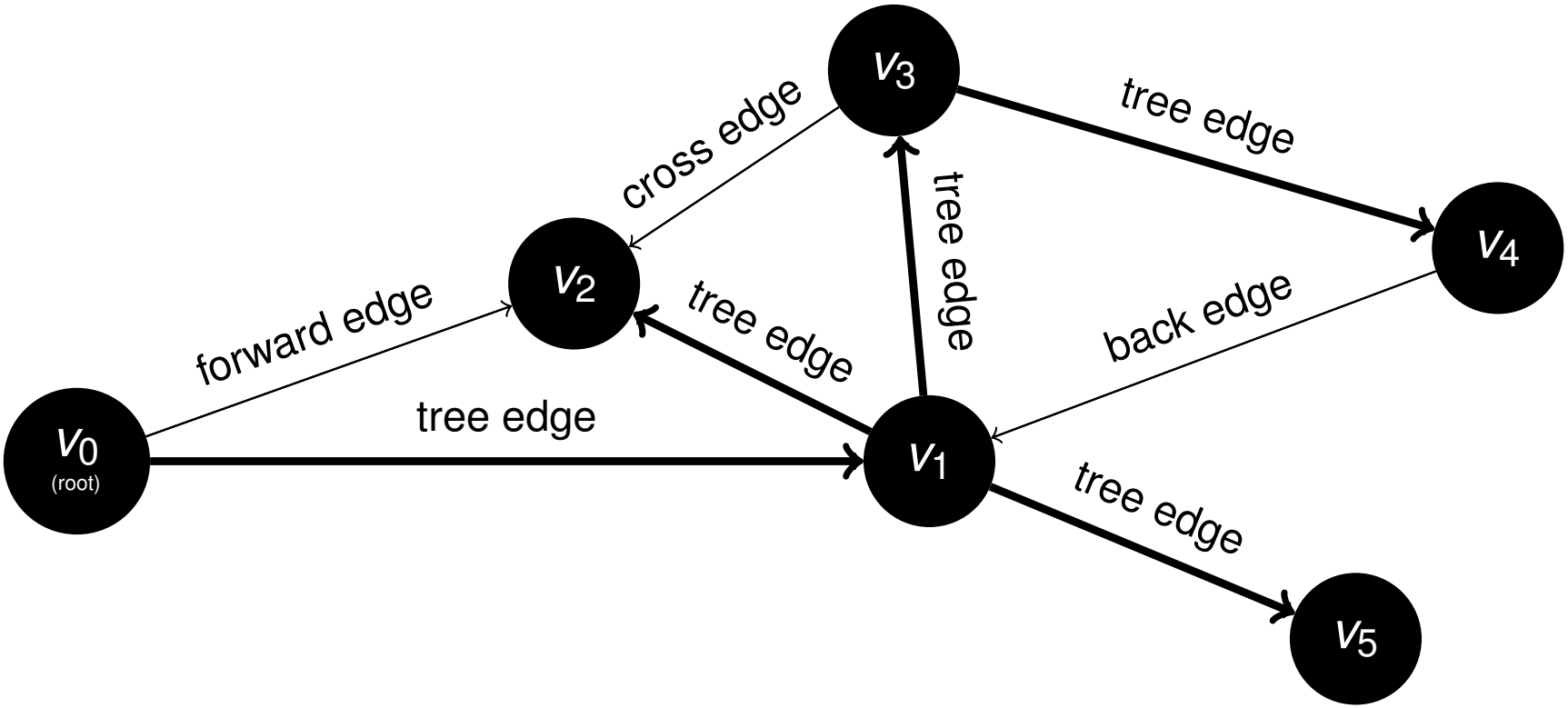

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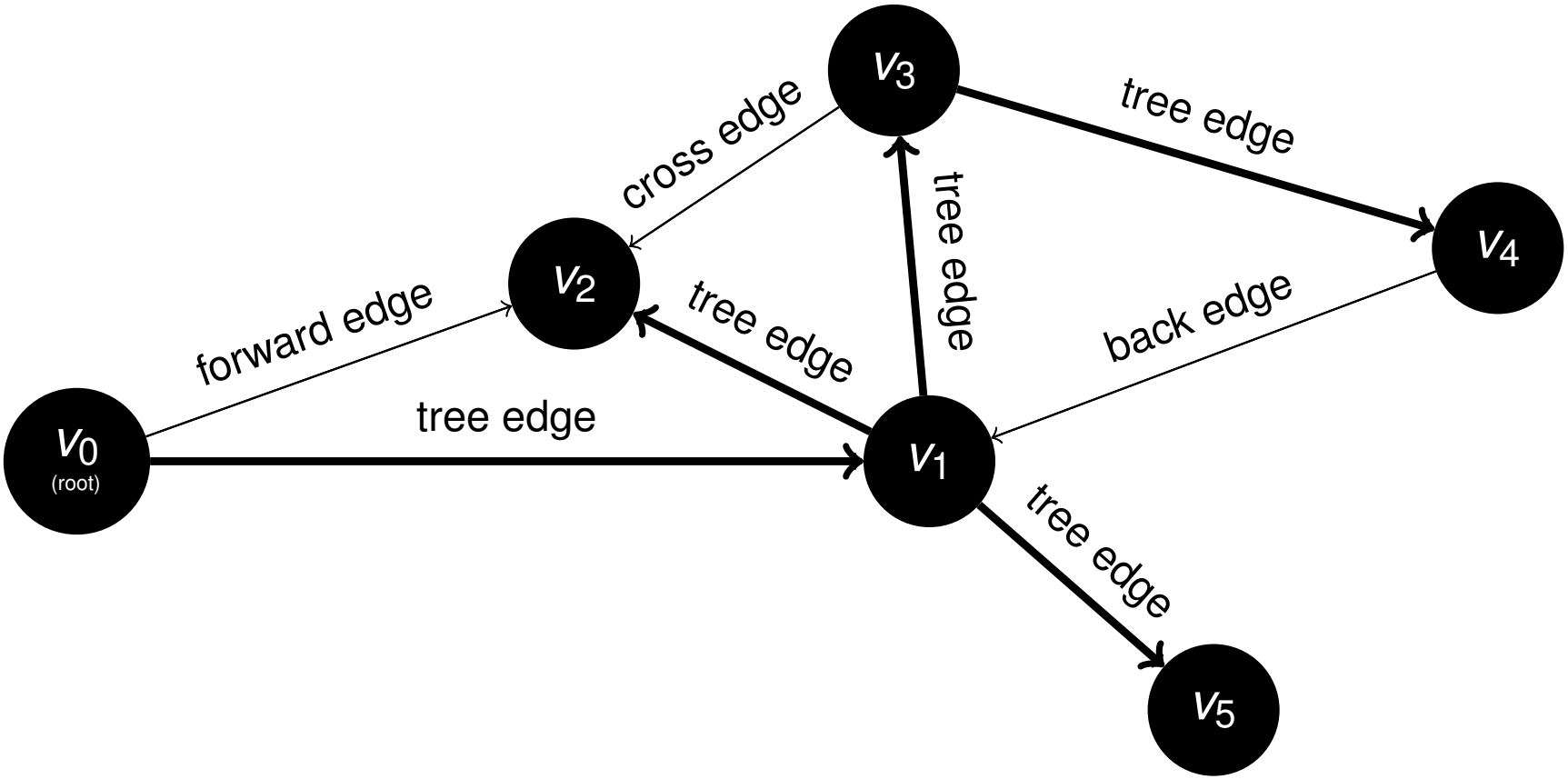

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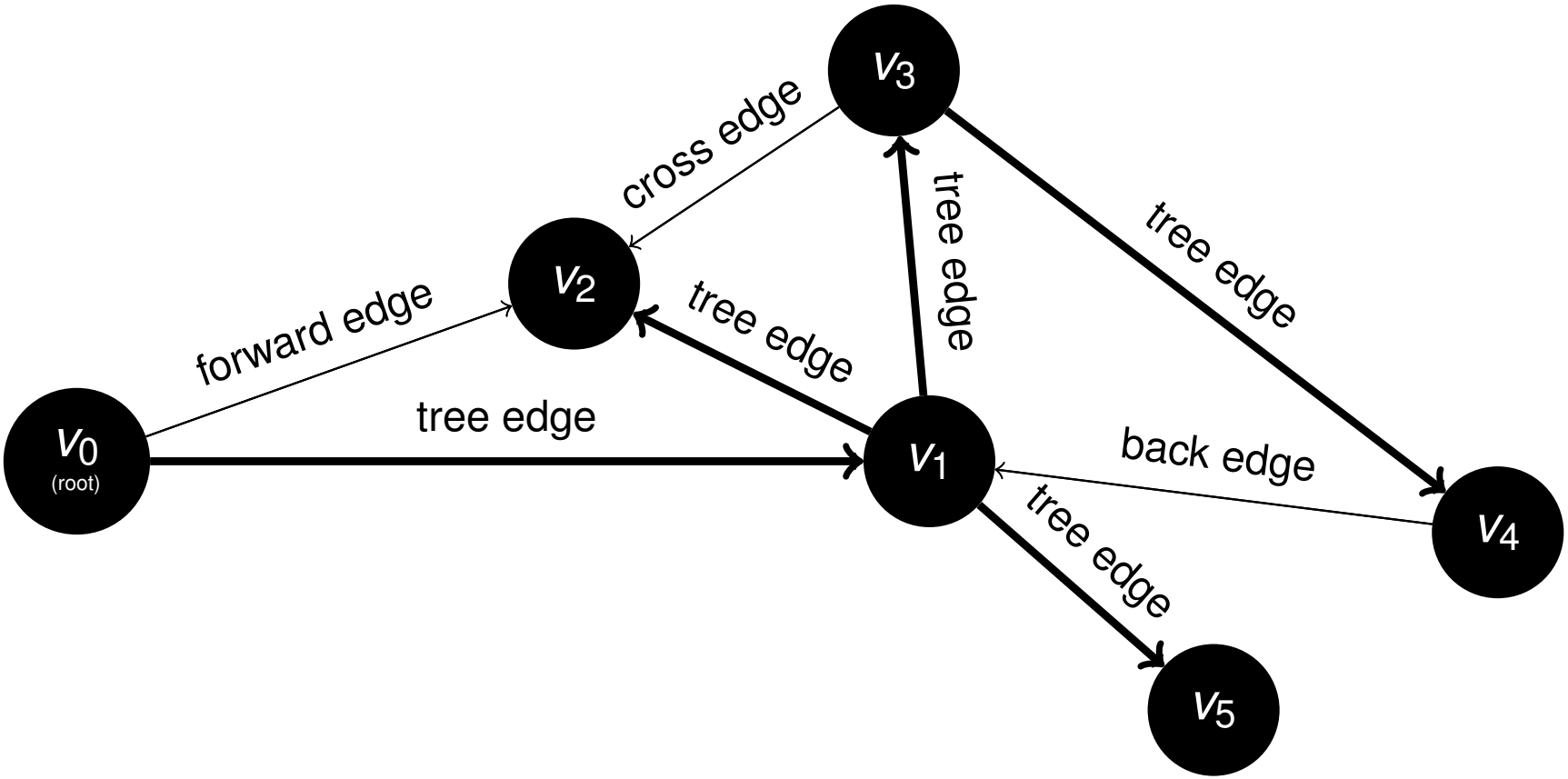

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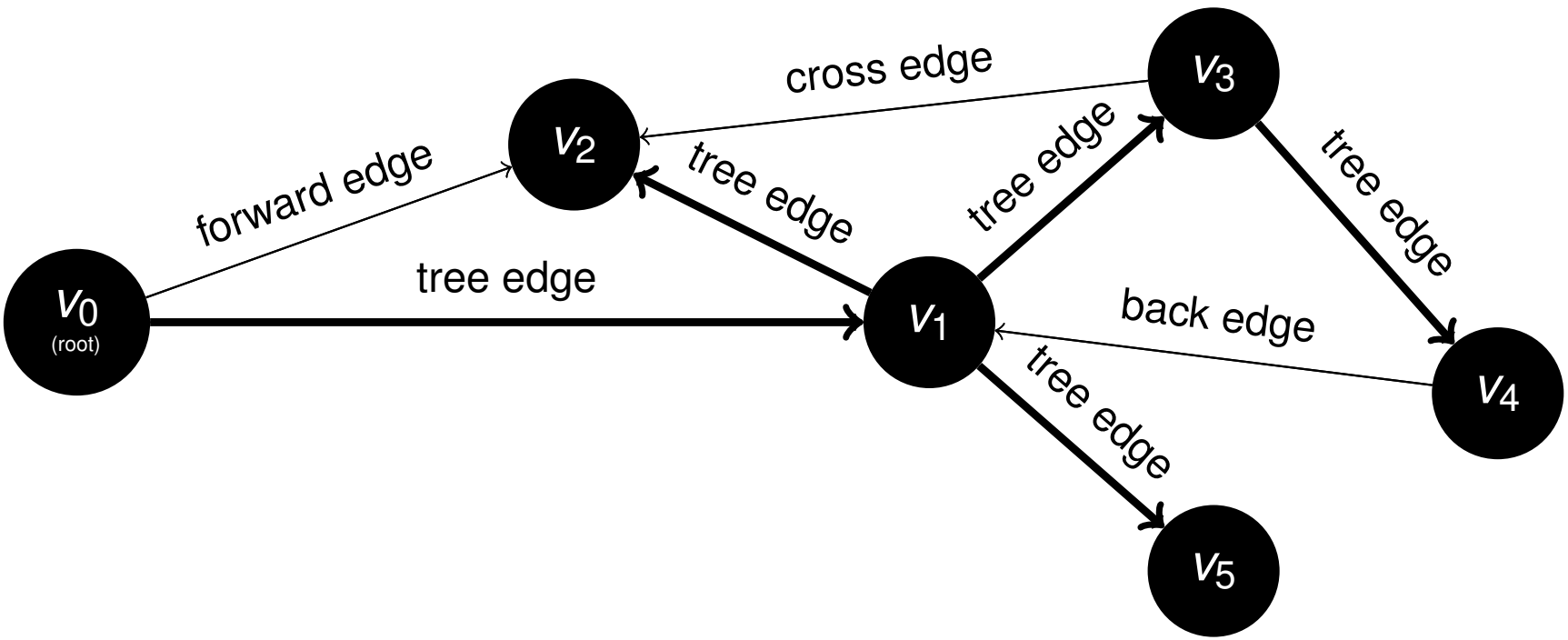

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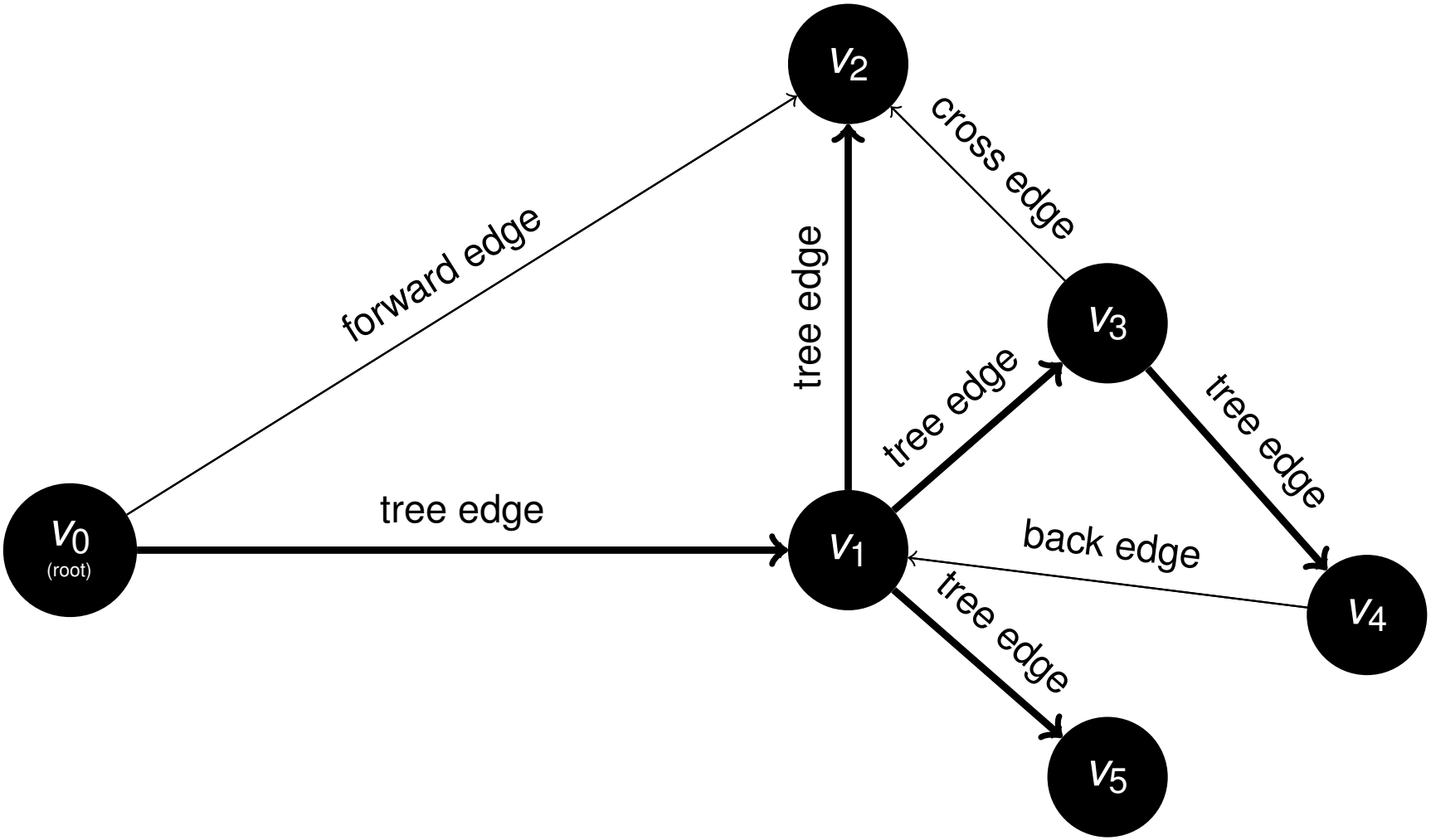

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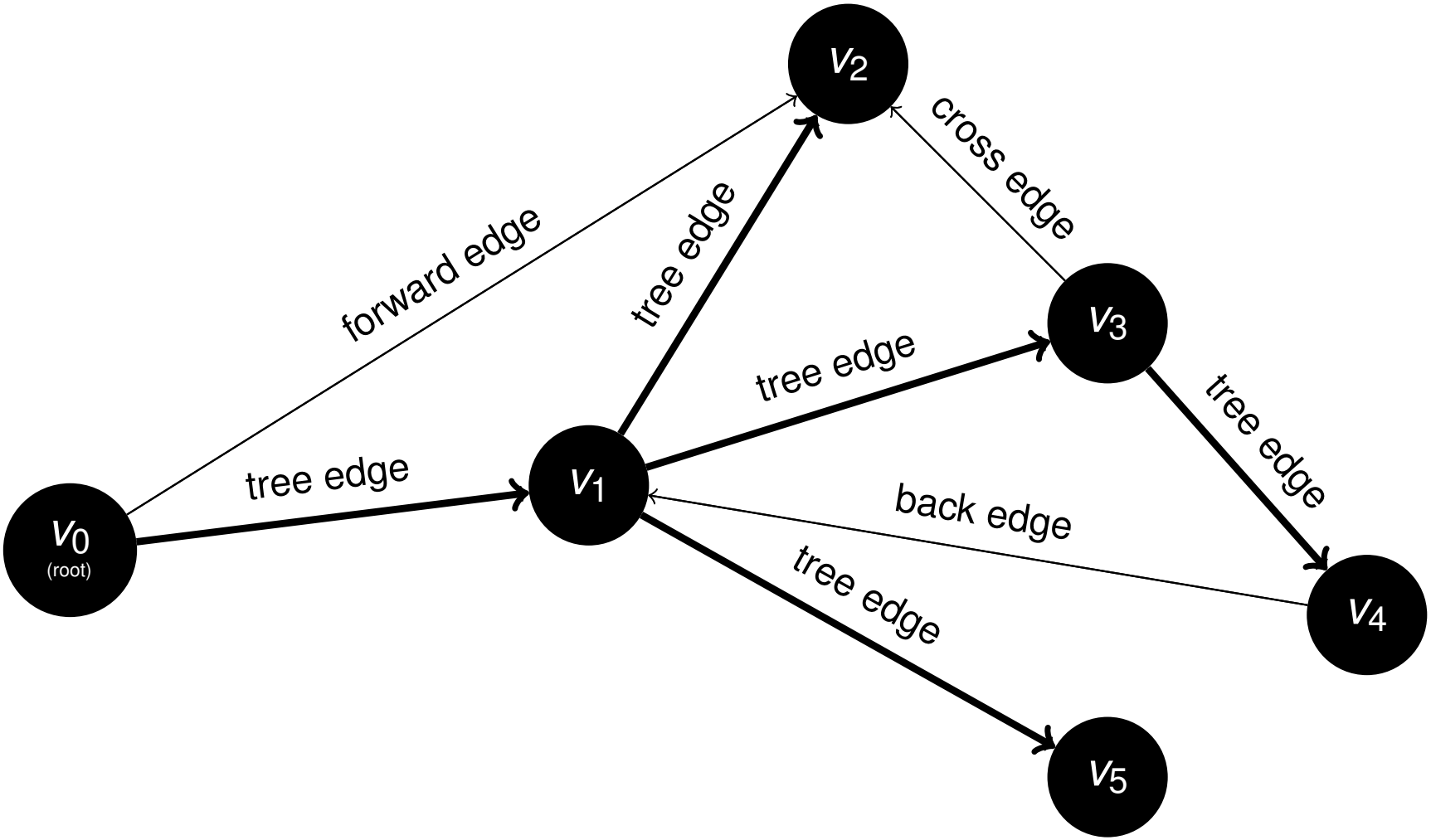

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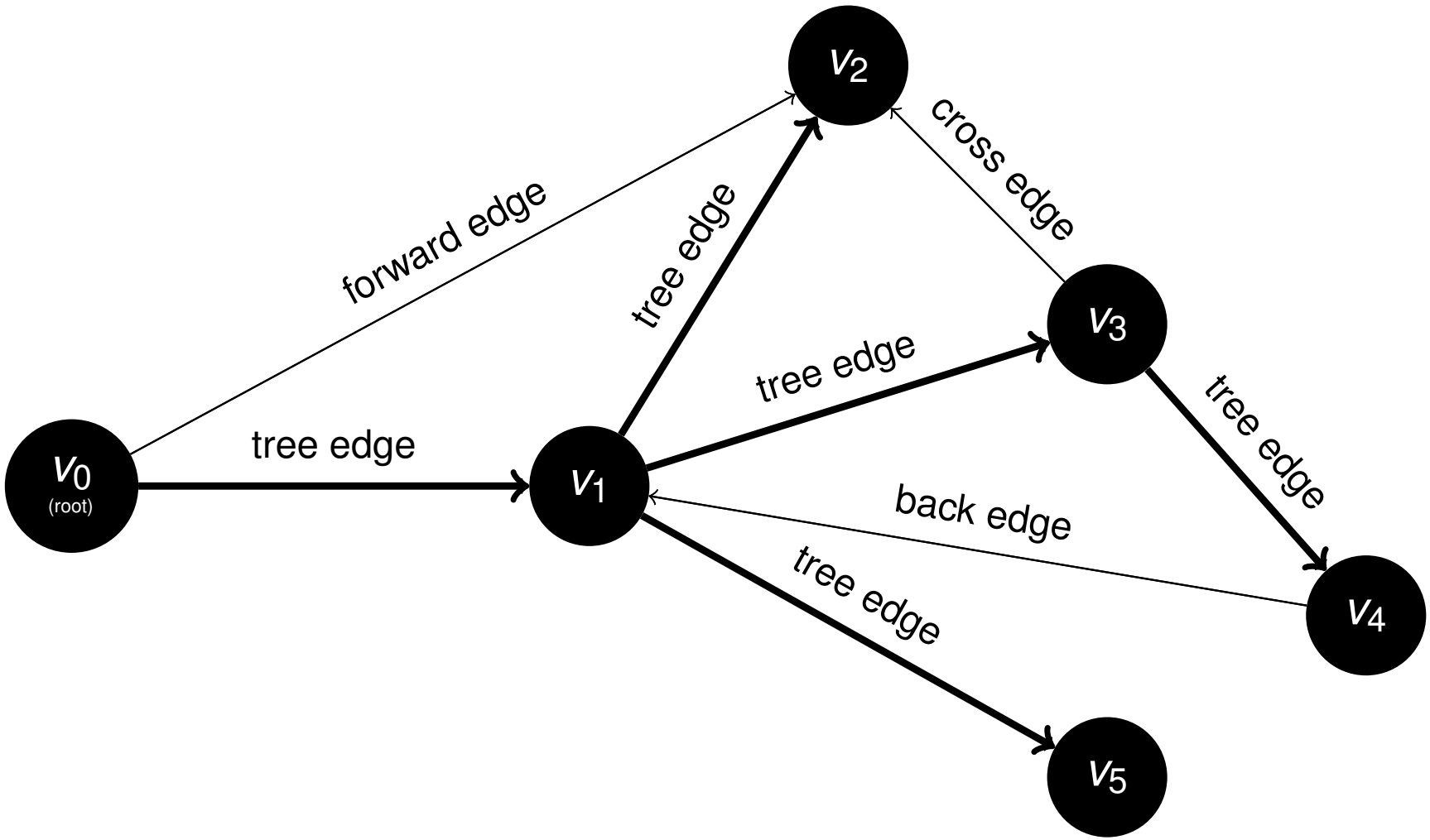

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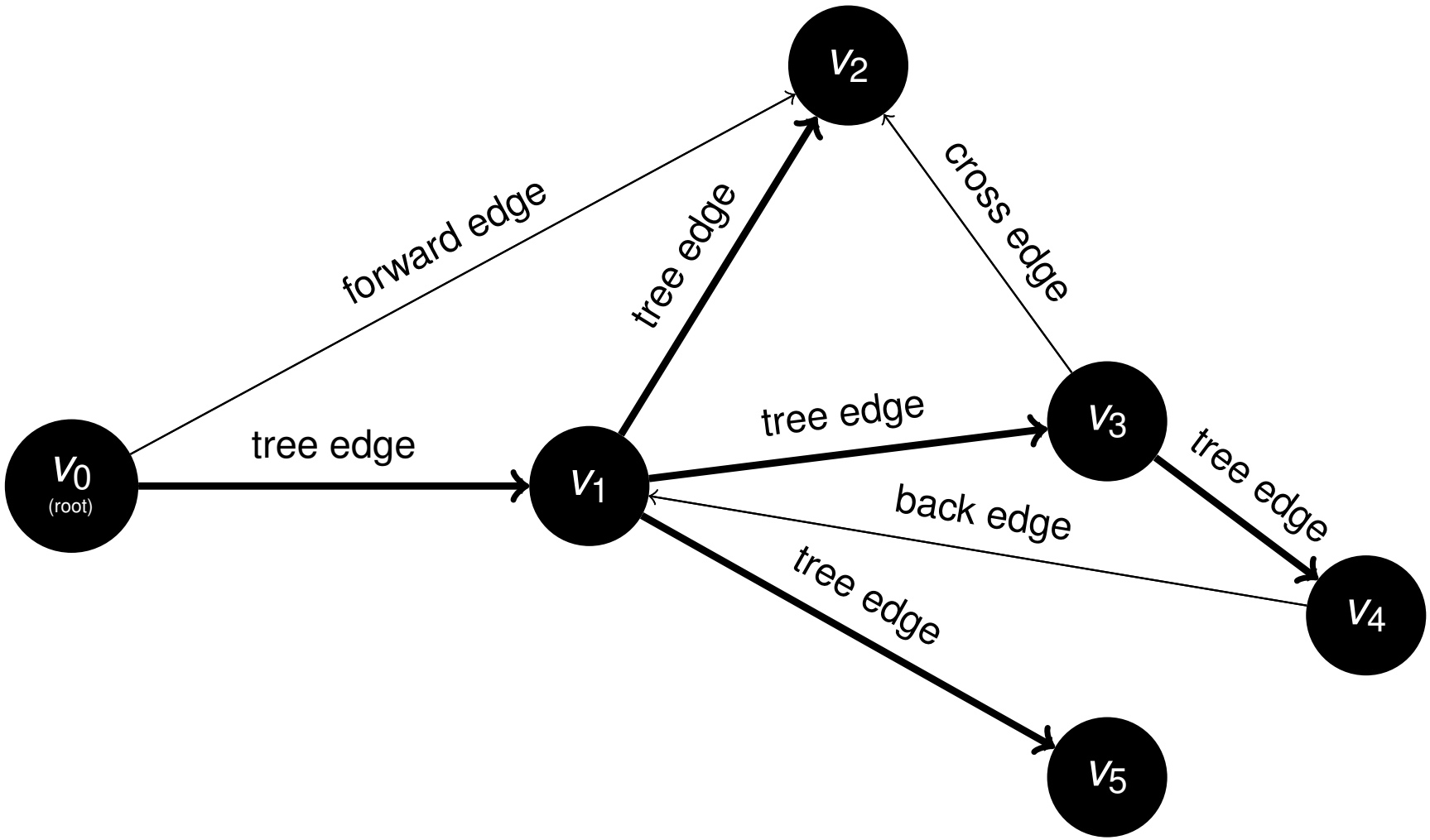

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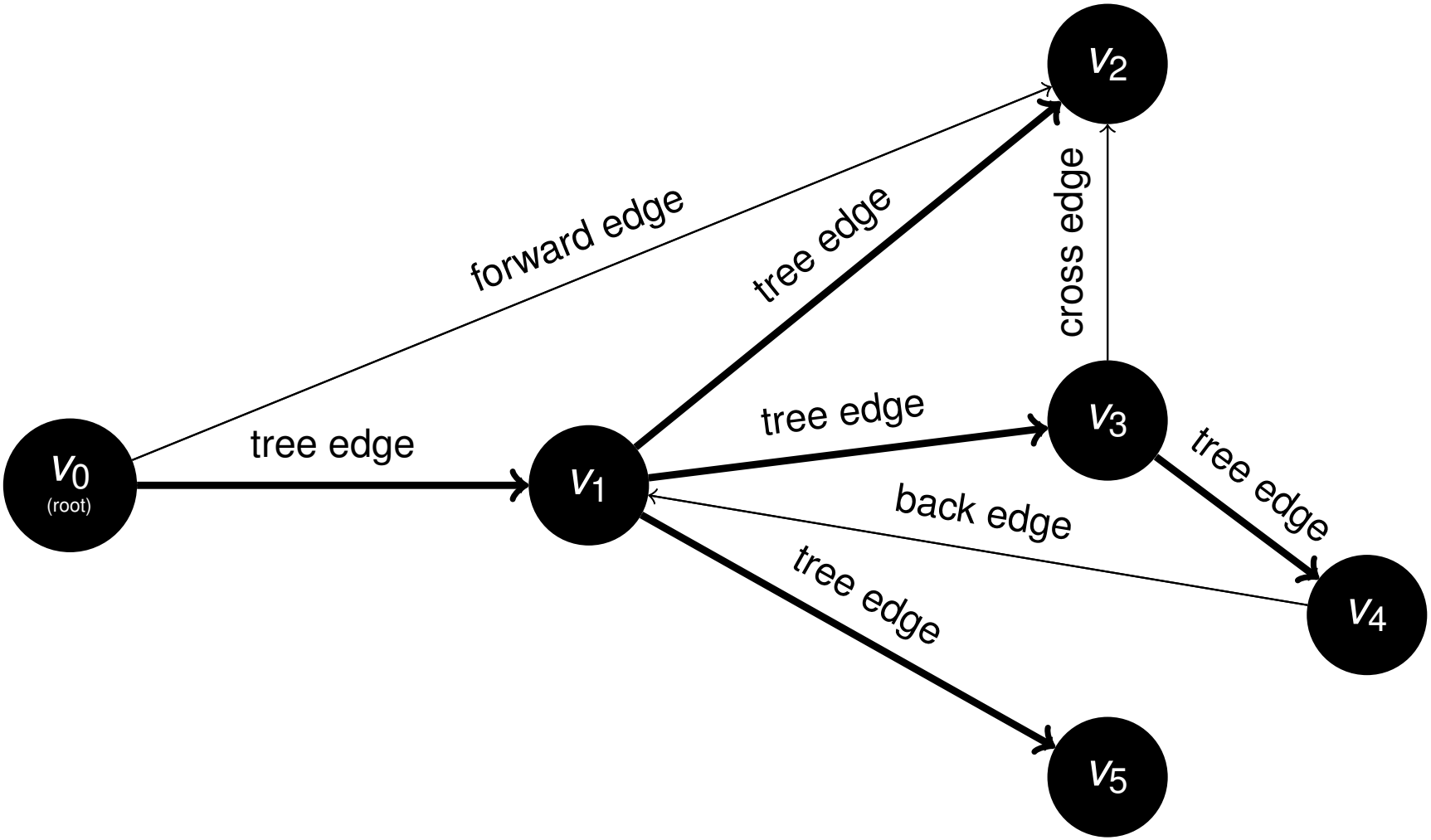

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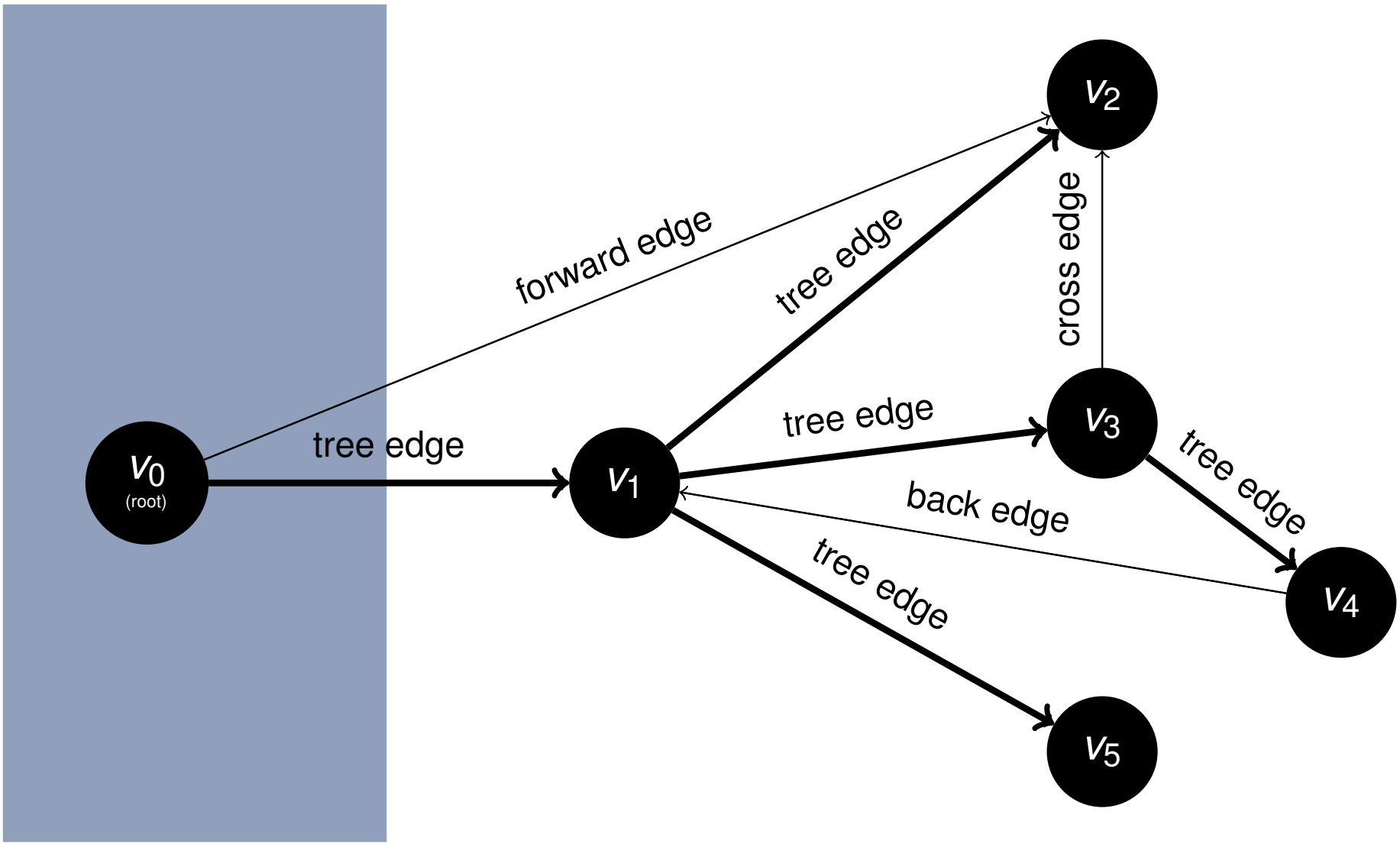

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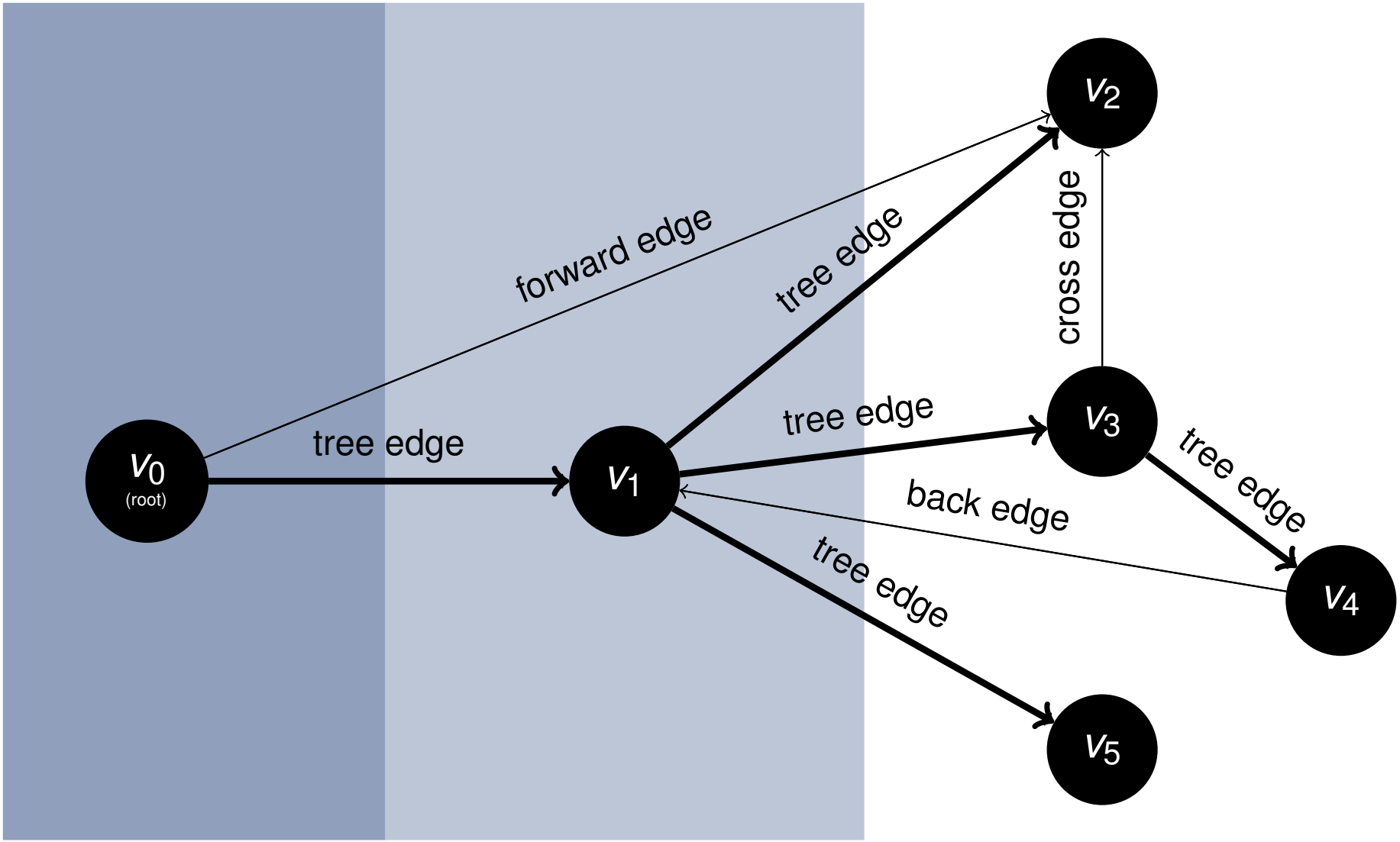

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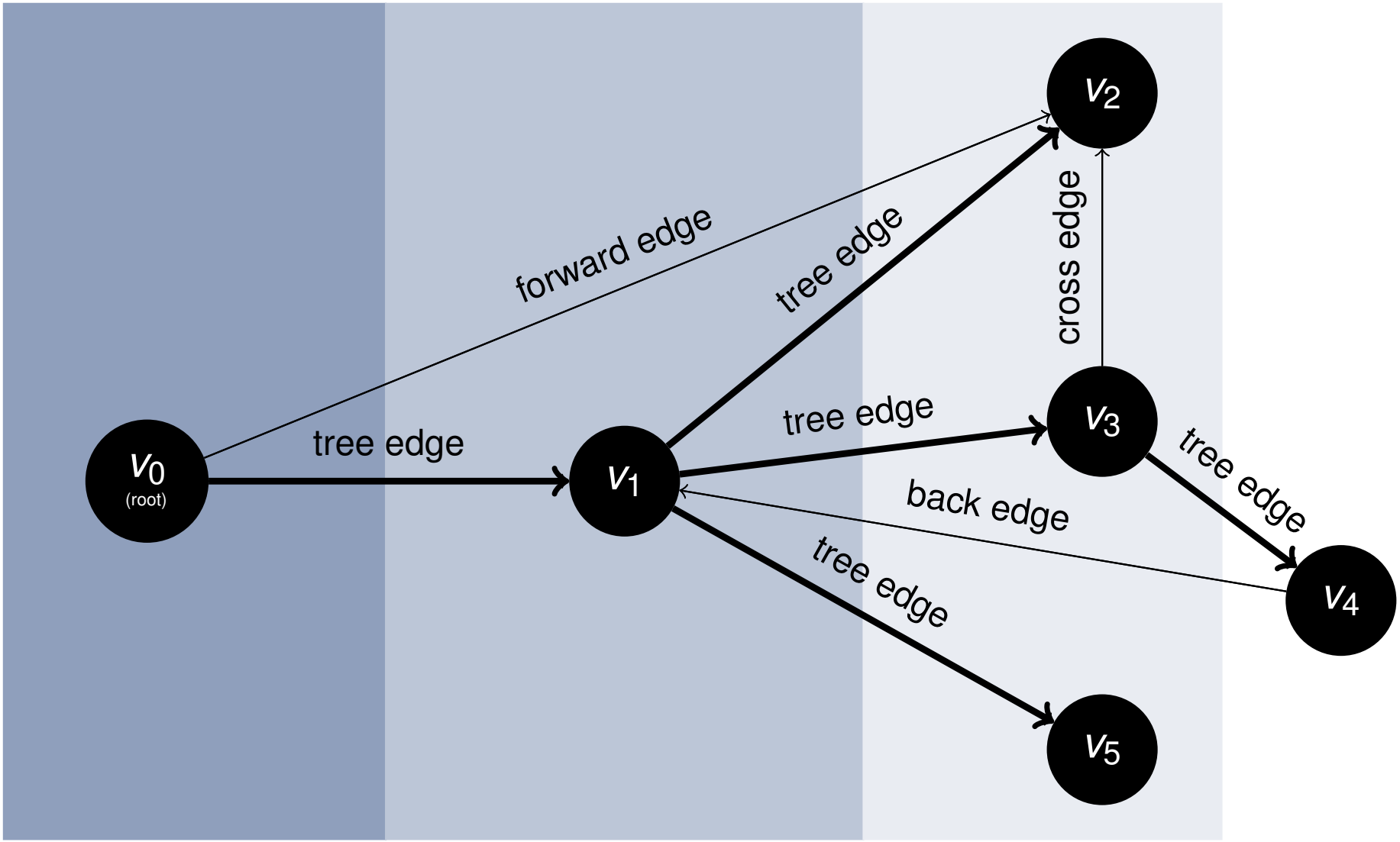

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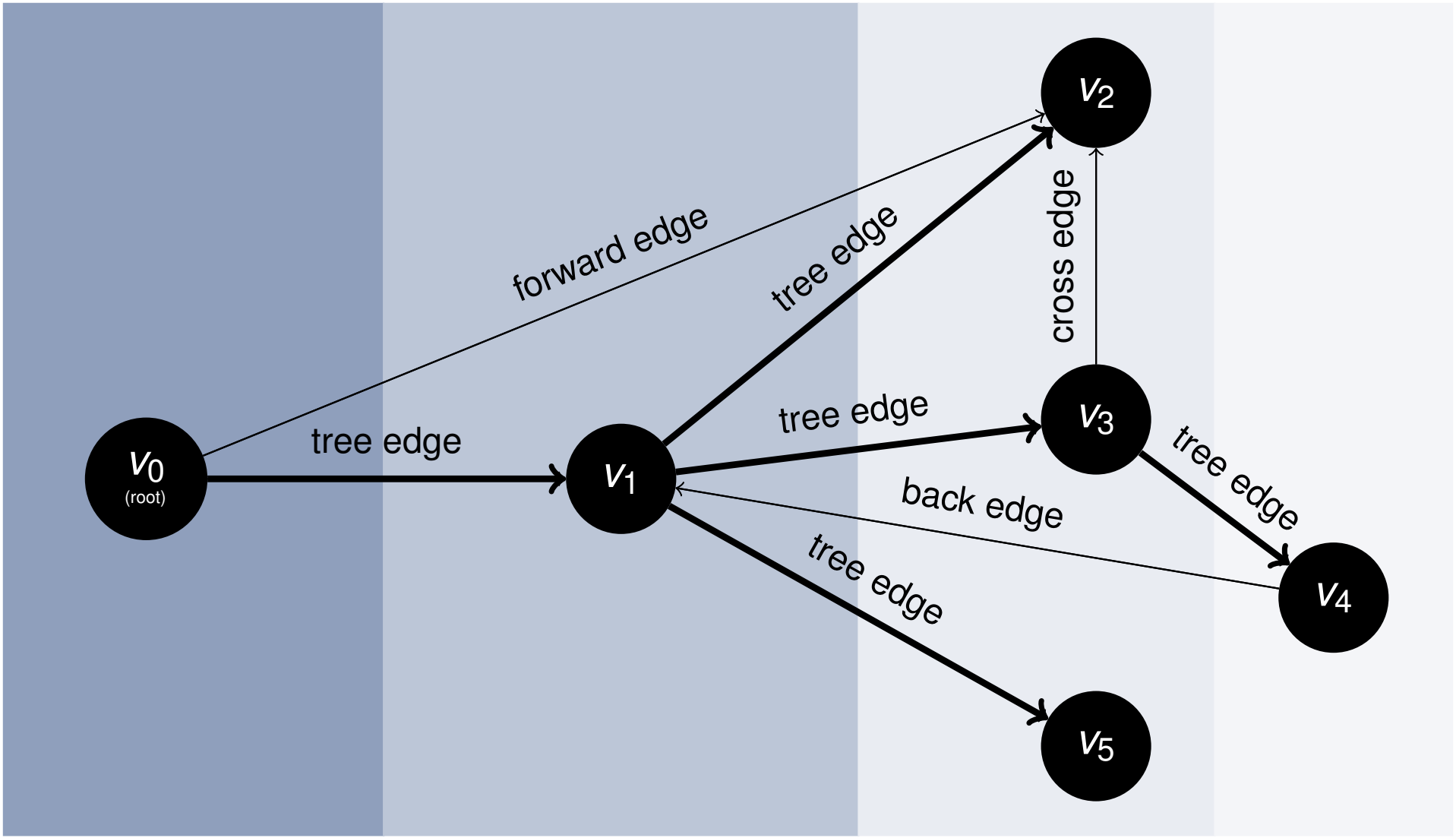

Finding Cycles

```
vector<vector<int>>
G { {1,2},
    {2,3,5},
    {},
    {2,4},
    {1},
    {} };
```

Finding Cycles

```
void cyclefind_dir(int current) {
   cout << current << " ";
   state[current] = 1;
   for (int i=0; i<G[current].size(); ++i) {
      int v = G[current][i];
      if (state[v] == 0) {
         cyclefind_dir(v);
      } else if (state[v] == 1) {
         cout << "BACK EDGE";
      }
   }
   state[current] = 2;
}
```

Topological Sorting

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- Consider **directed acyclic graph** (**DAG**)

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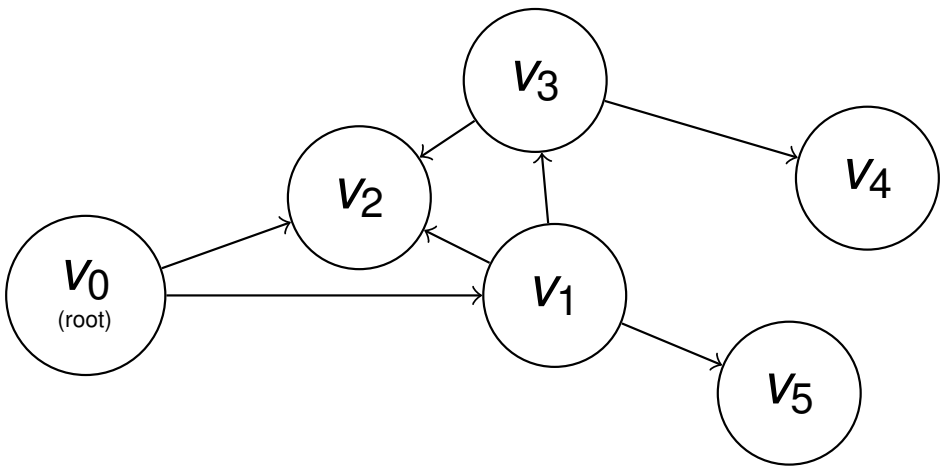

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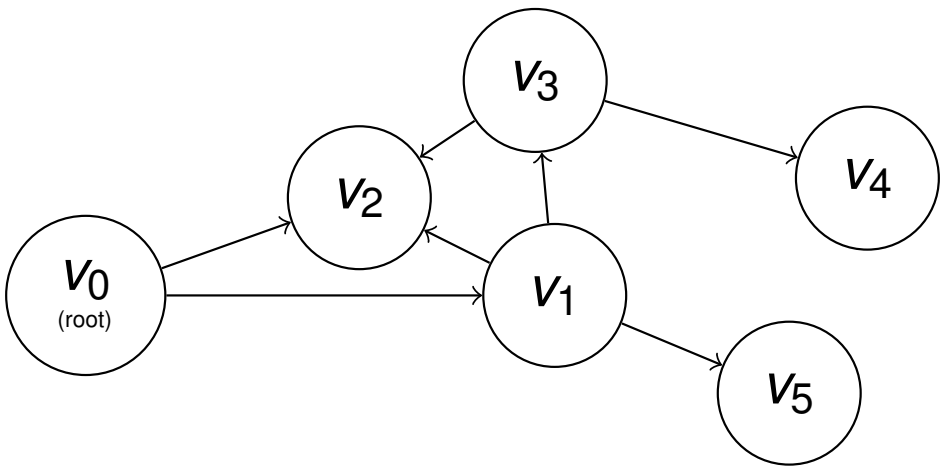

$$v_2 < v_4 < v_3 < v_5 < v_1 < v_0$$

Topological Sorting

```
vector<vector<int>>
G { {1,2},
    {2,3,5},
    {},
    {2,4},
    {},
    {} };
```

Topological Sorting

- Use DFS

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- After recursive call, all “smaller” vertices have been visited

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- Then push `current` to back of stack

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- After recursive call, all “smaller” vertices have been visited
- Then push `current` to back of stack

```
void toposort(int current) {
   state[current] = 1;
   for (int i=0; i<G[current].size(); ++i) {
      int v = G[current][i];
      if (state[v] == 0) {
         toposort(v);
      }
   }
   state[current] = 2;
   topo.push_back(current);
}
```

Cheese Patrol

Cheese Patrol

- Surveillance of Tokyo

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- Detectives can be placed on streets and intersections

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- When on intersection, detectives surveils two streets

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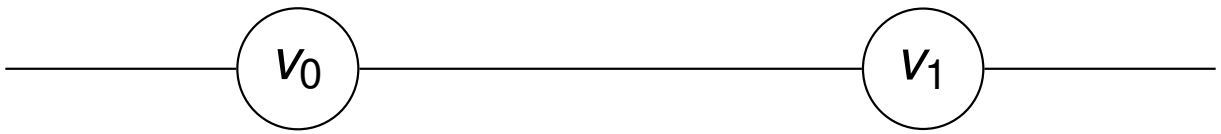

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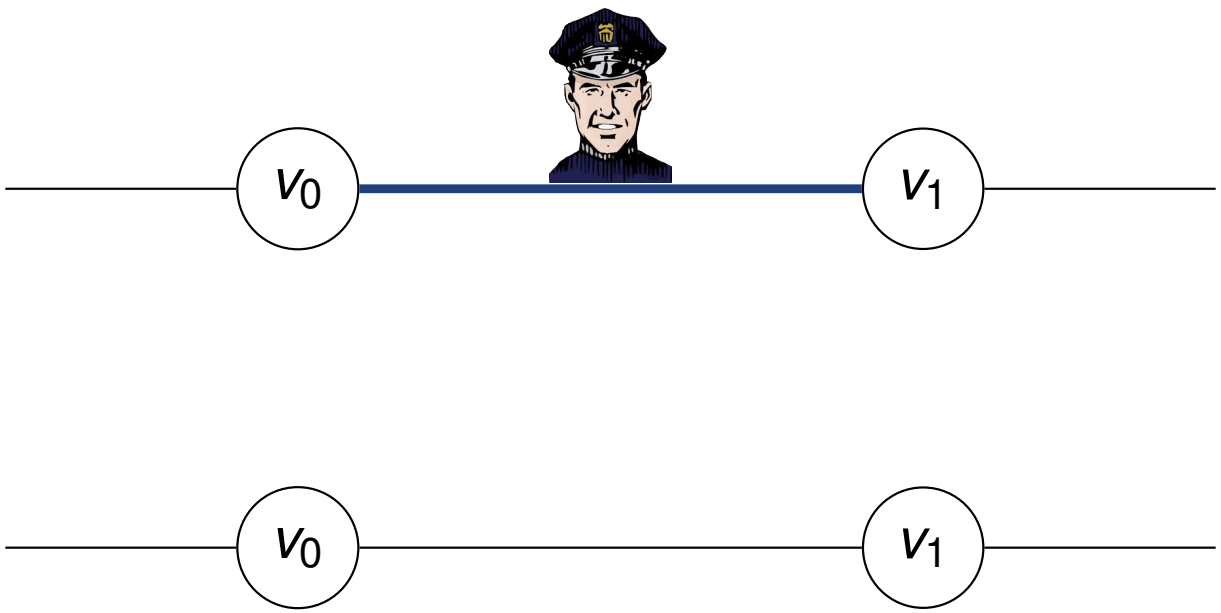

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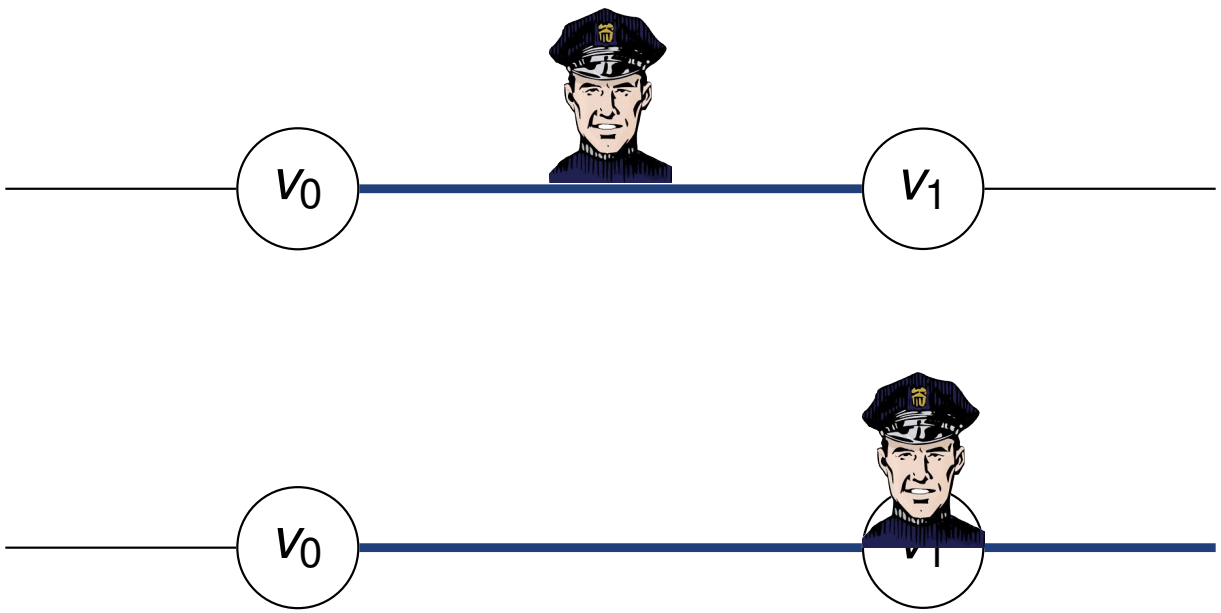

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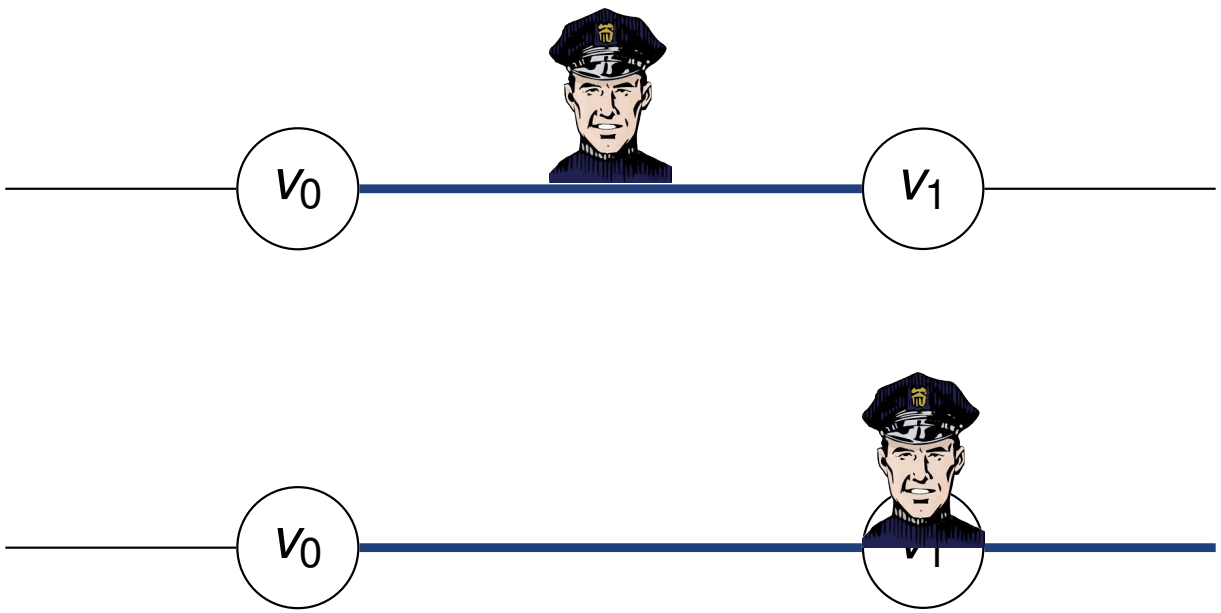

Surveil complete town with minimum number of detectives

Cheese Patrol

What is the best anyone can do?

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$$\sum_{i=1}^{\ell} \lceil m_i/2 \rceil \text{ detectives}$$

Cheese Patrol

Observations

- It is never superior to place detective at street

Cheese Patrol

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- Only place detectives at intersections
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- In each component, there may be one exception

Cheese Patrol

Assume graph is a tree

Cheese Patrol

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Careful about leaves

Cheese Patrol

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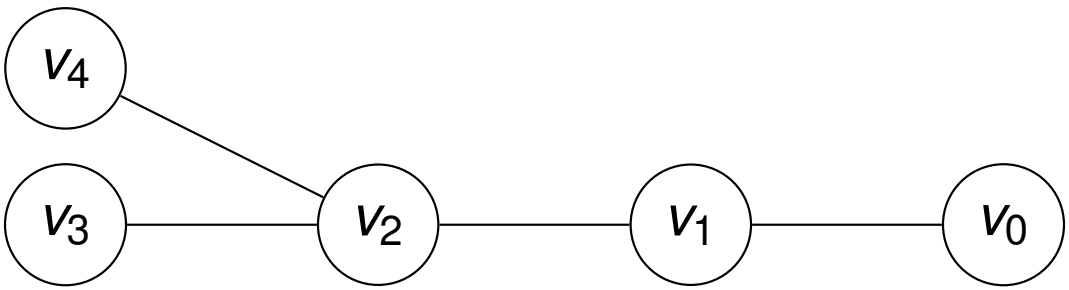

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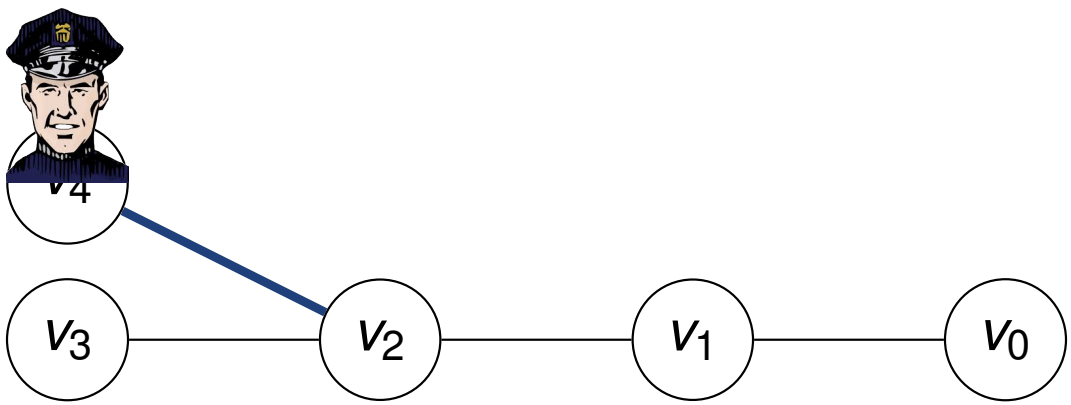

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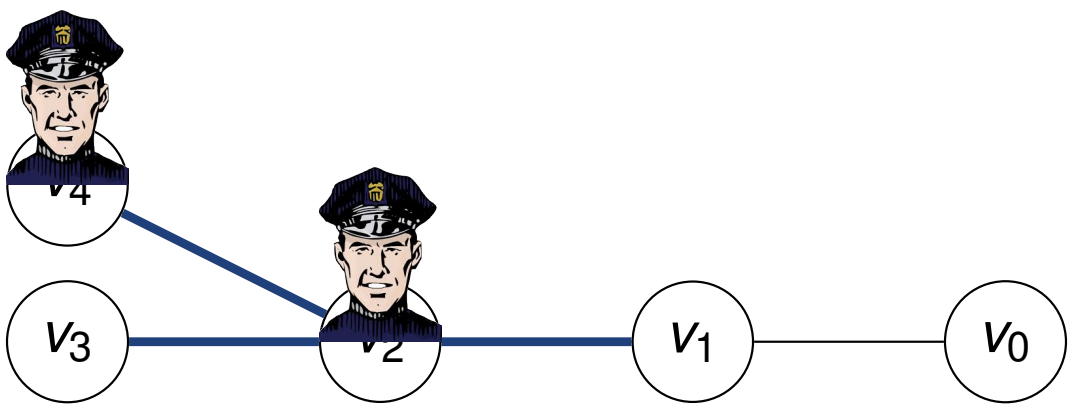

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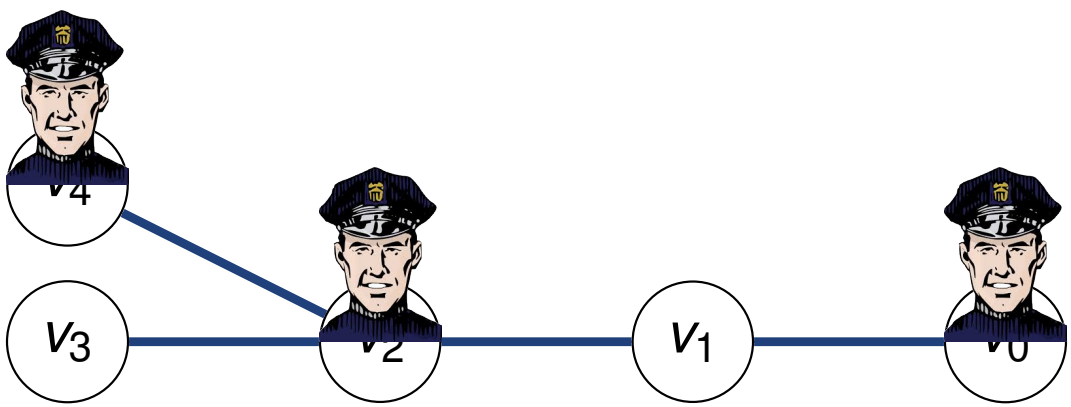

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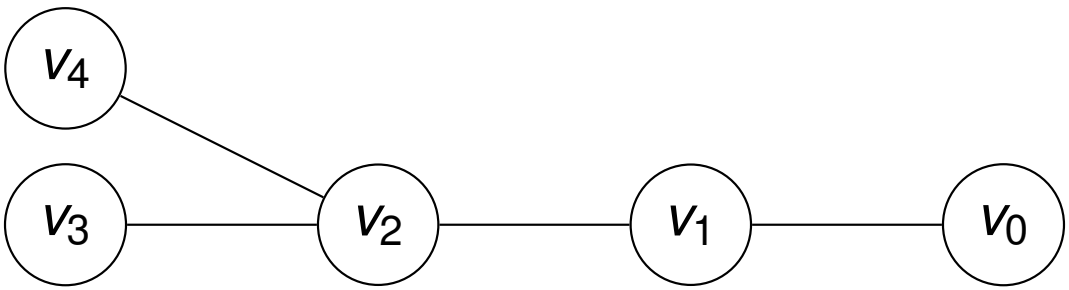

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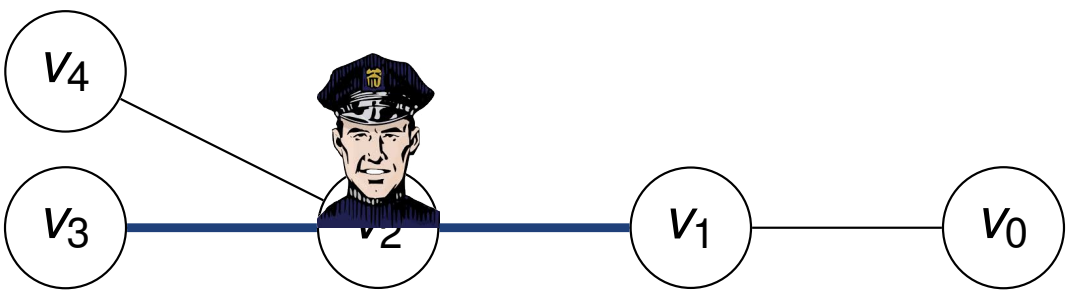

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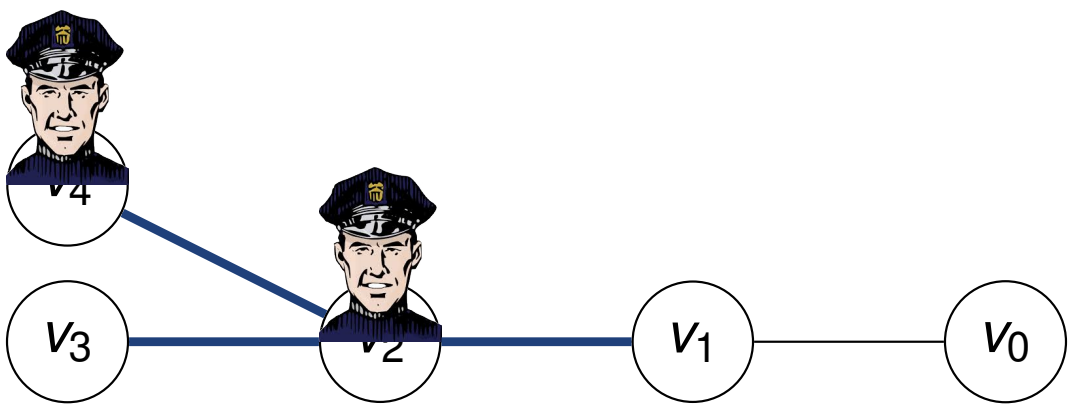

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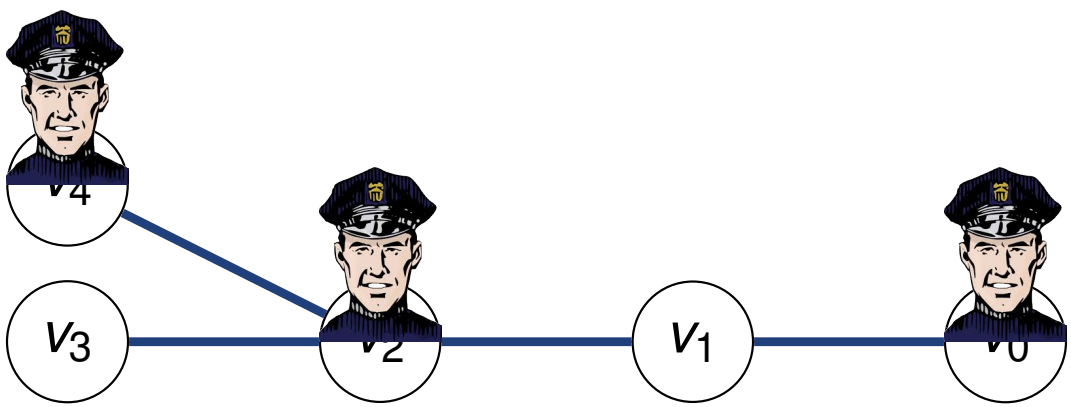

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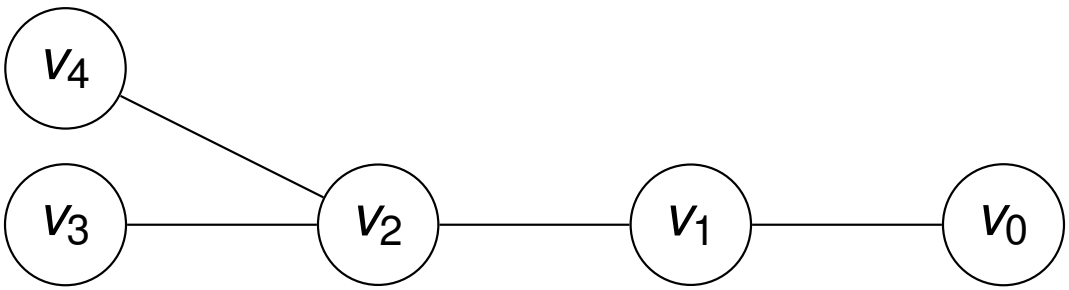

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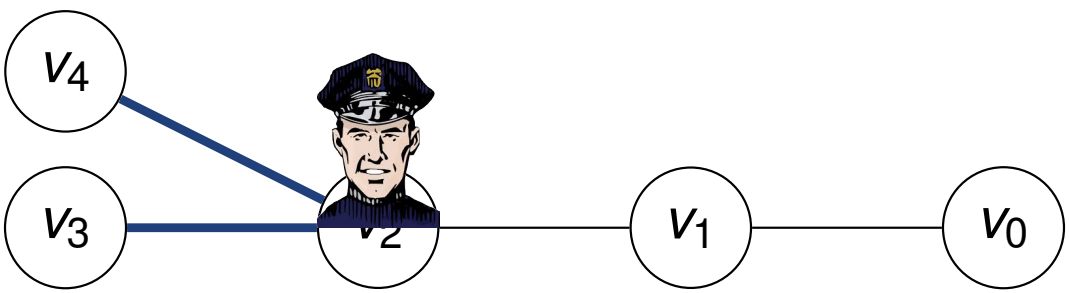

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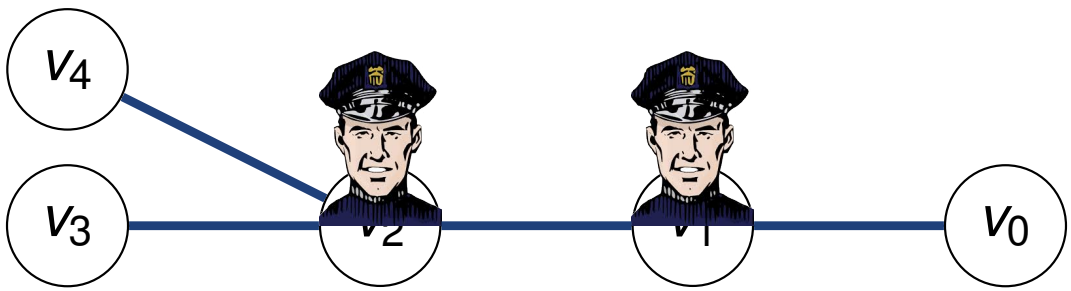

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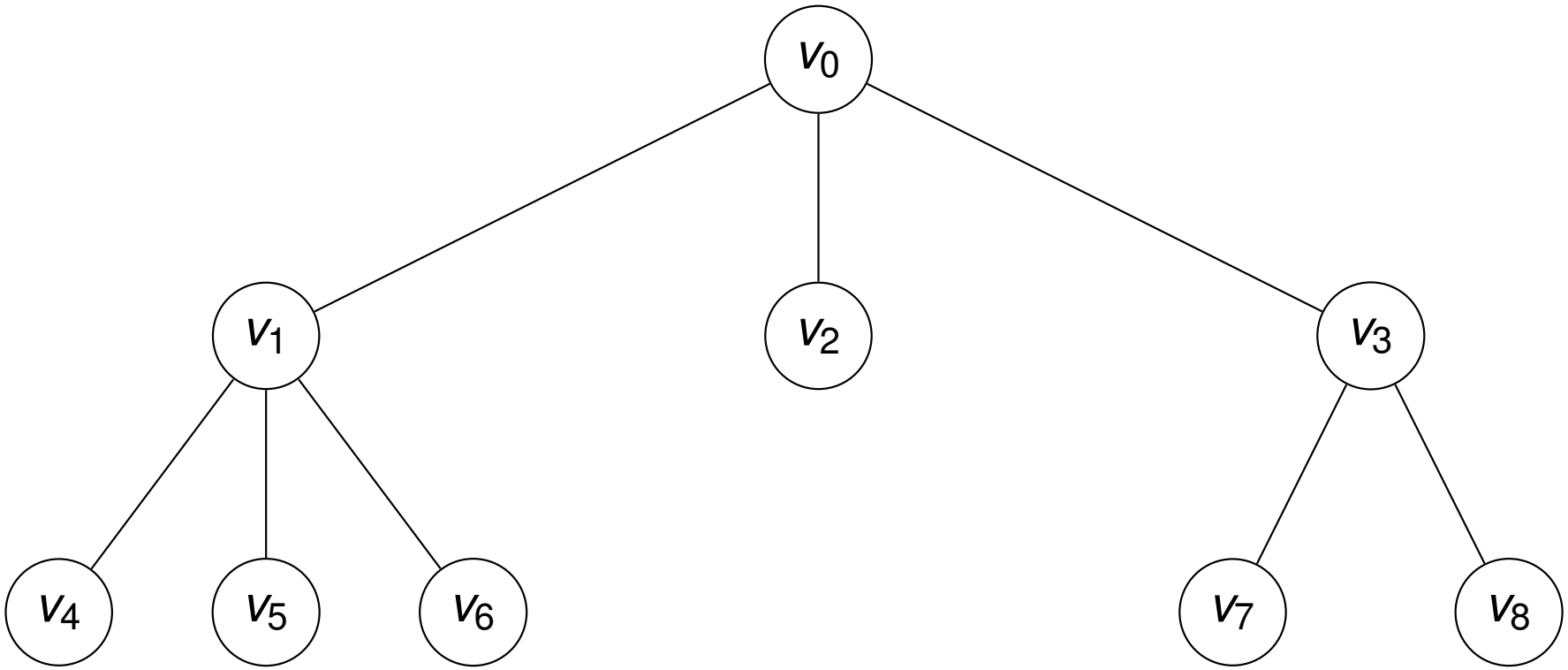

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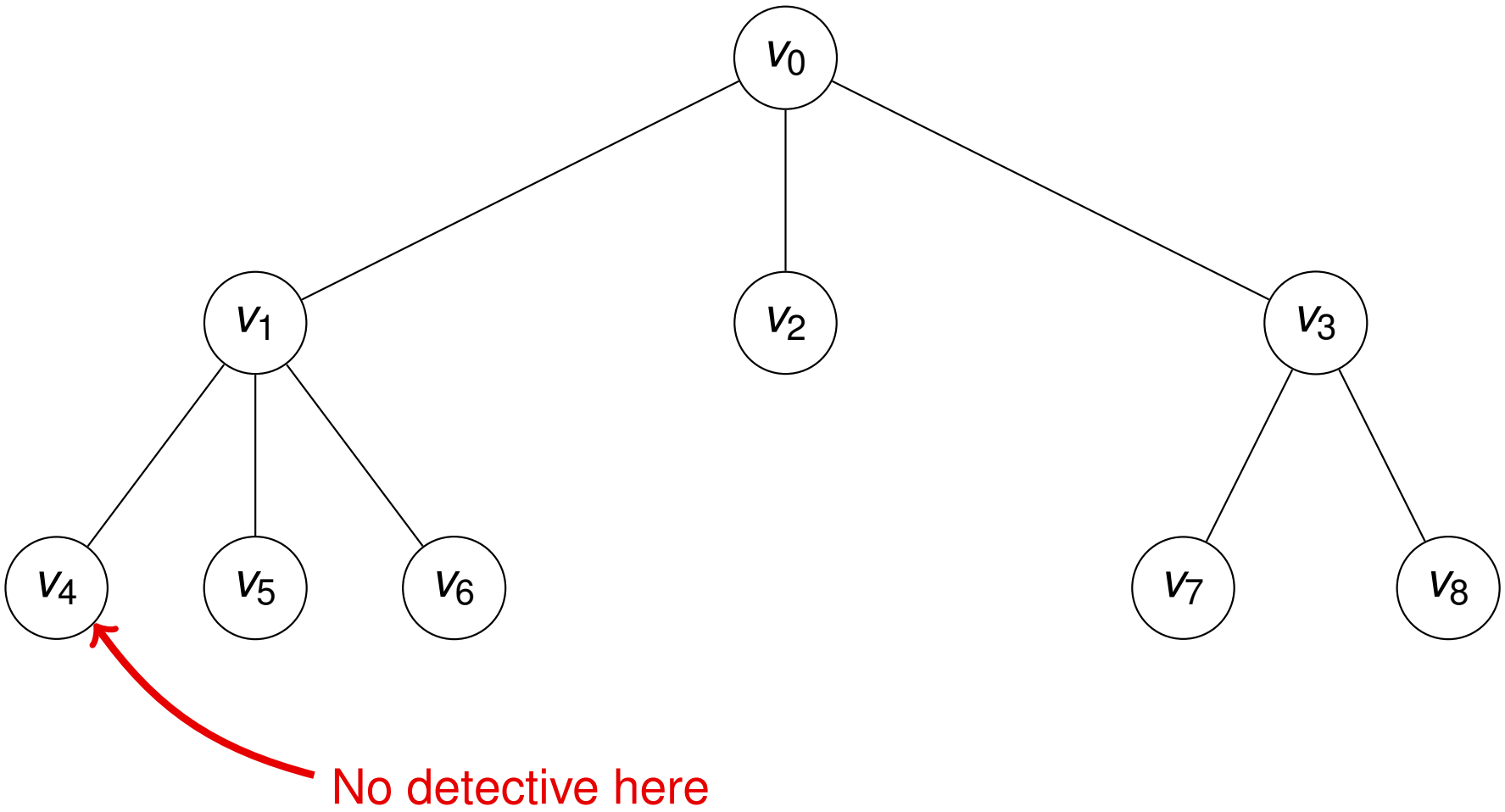

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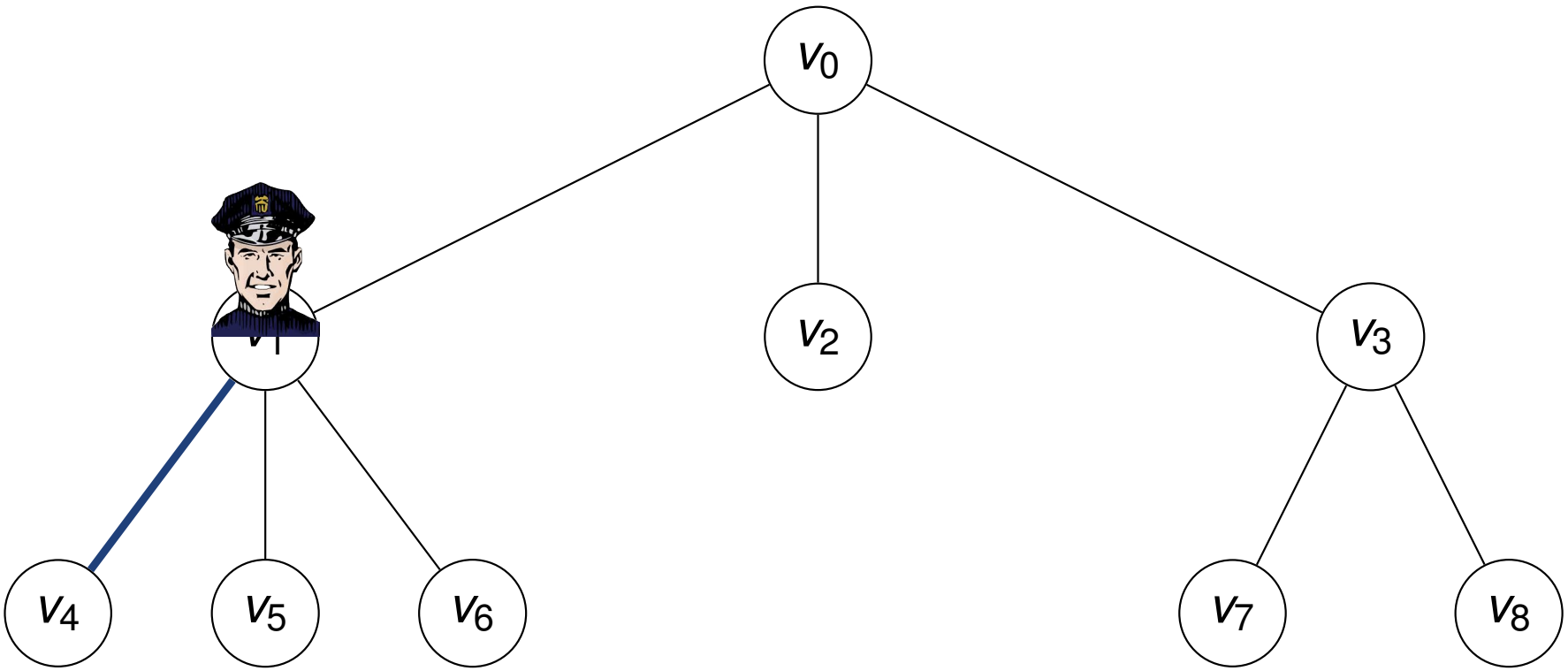

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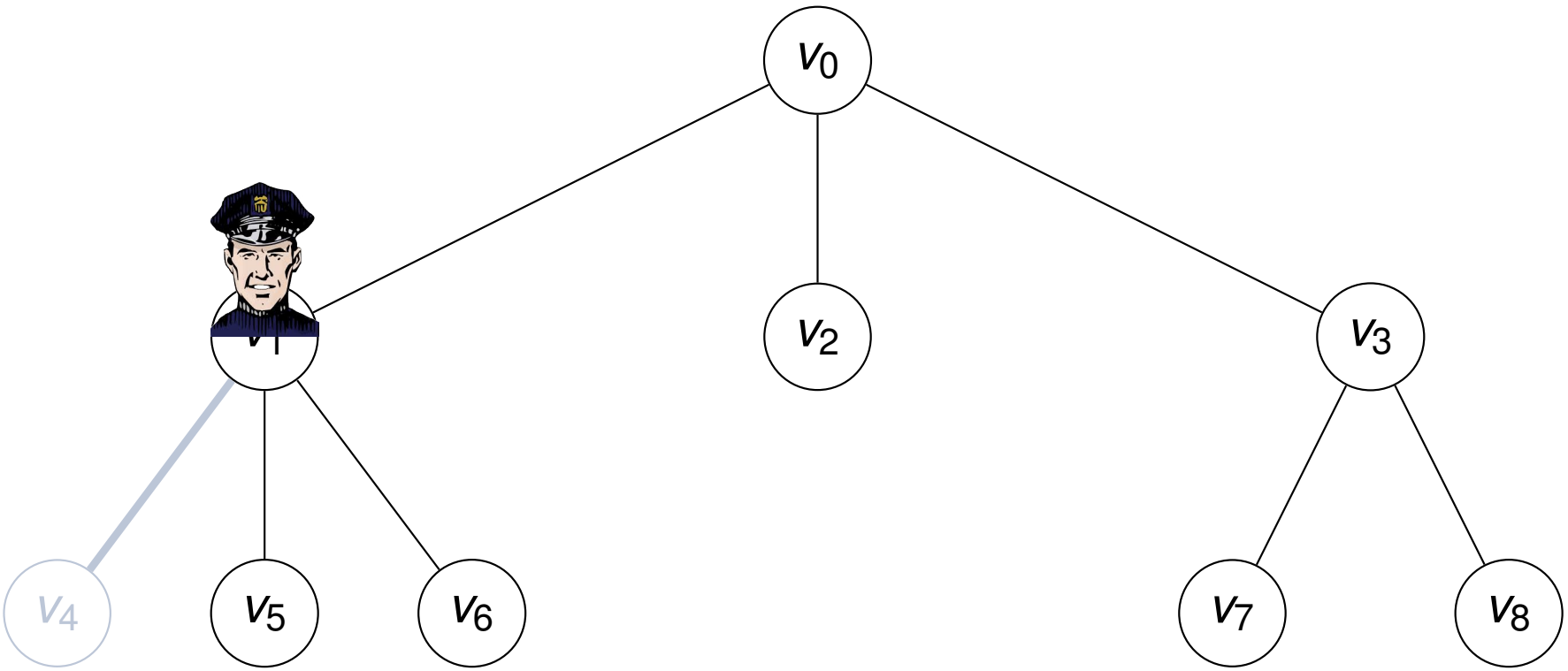

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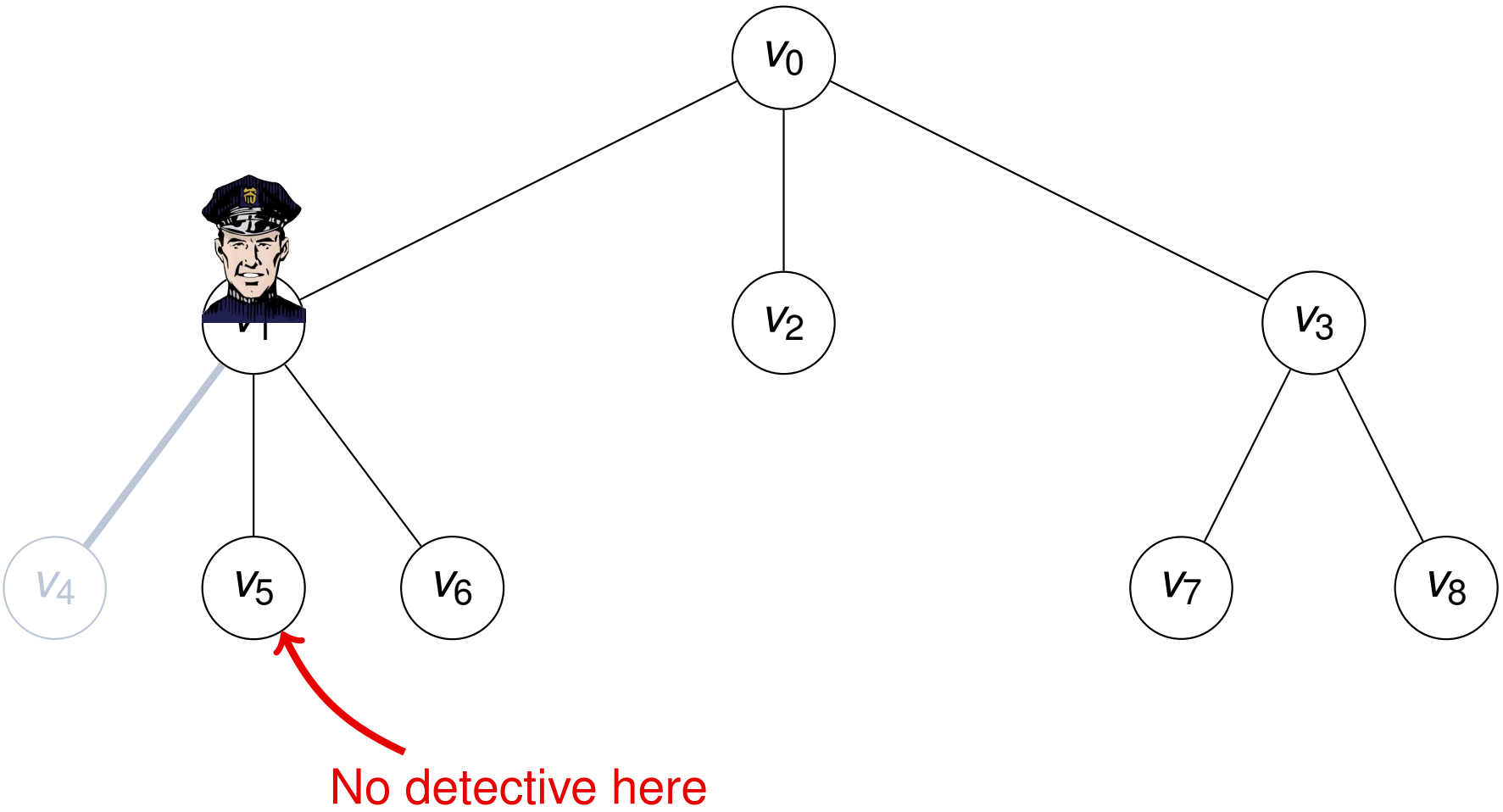

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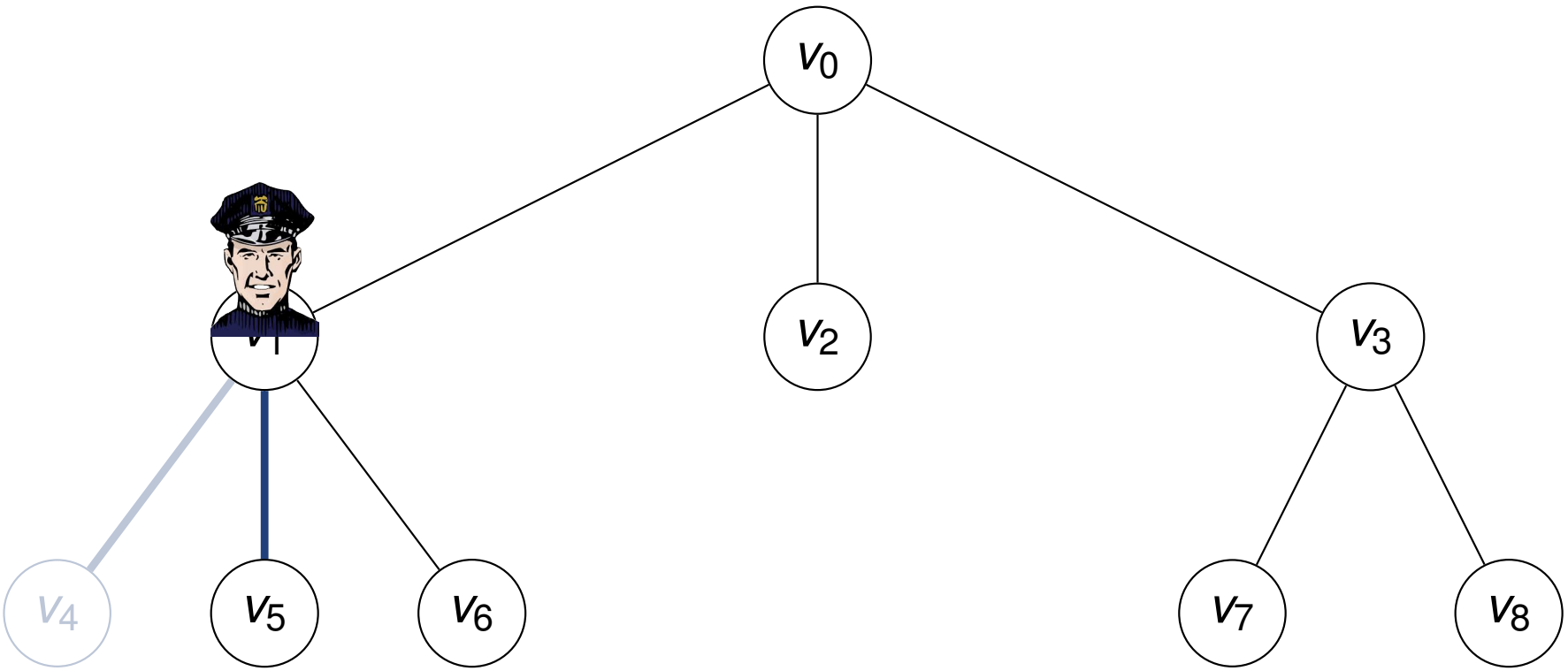

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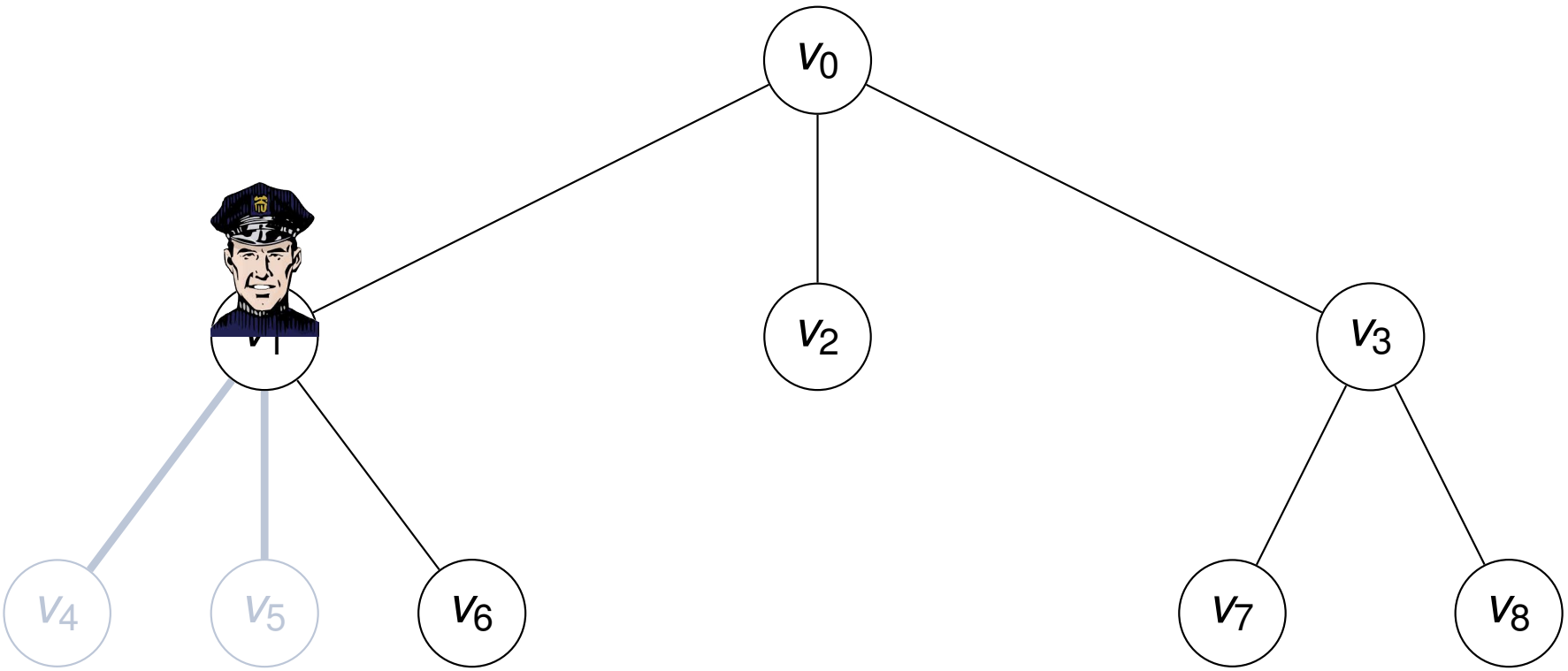

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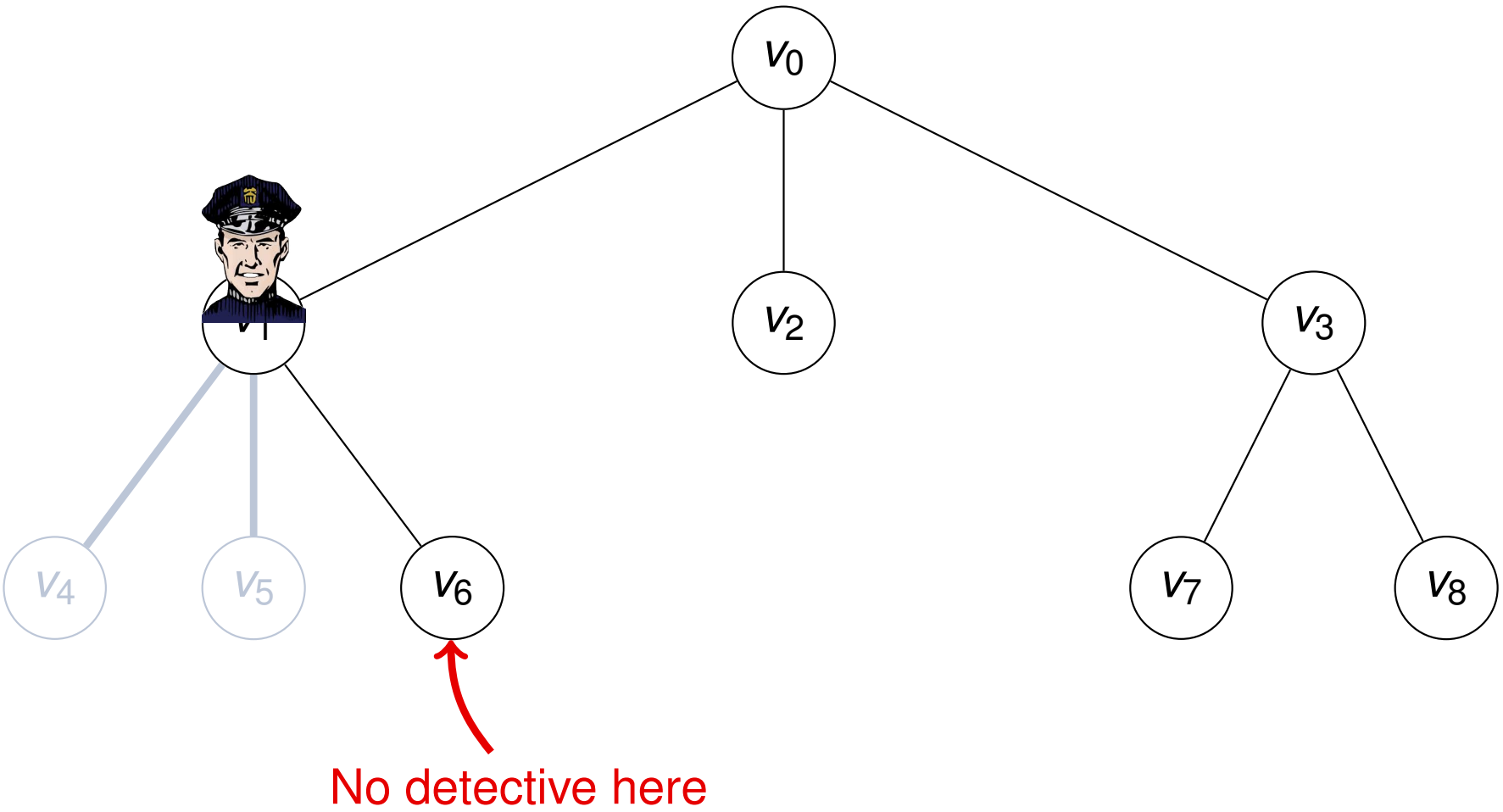

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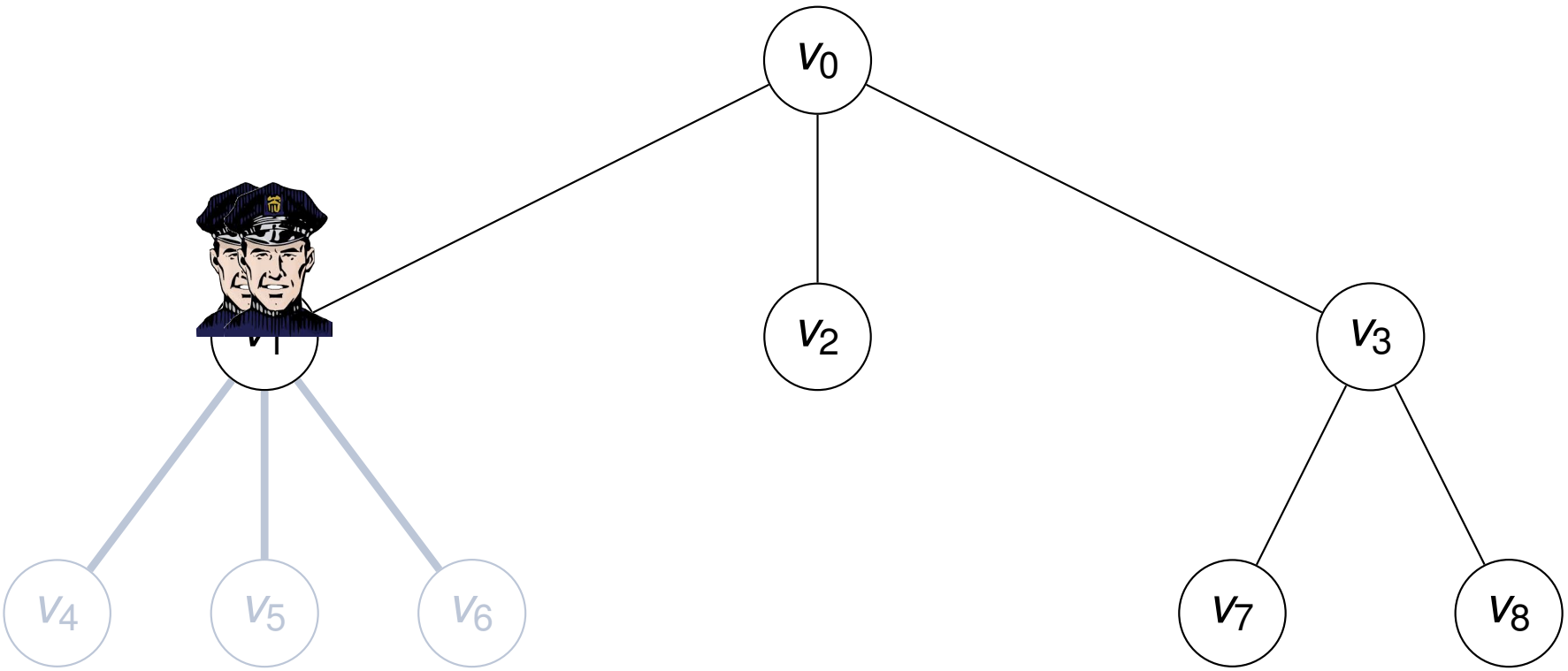

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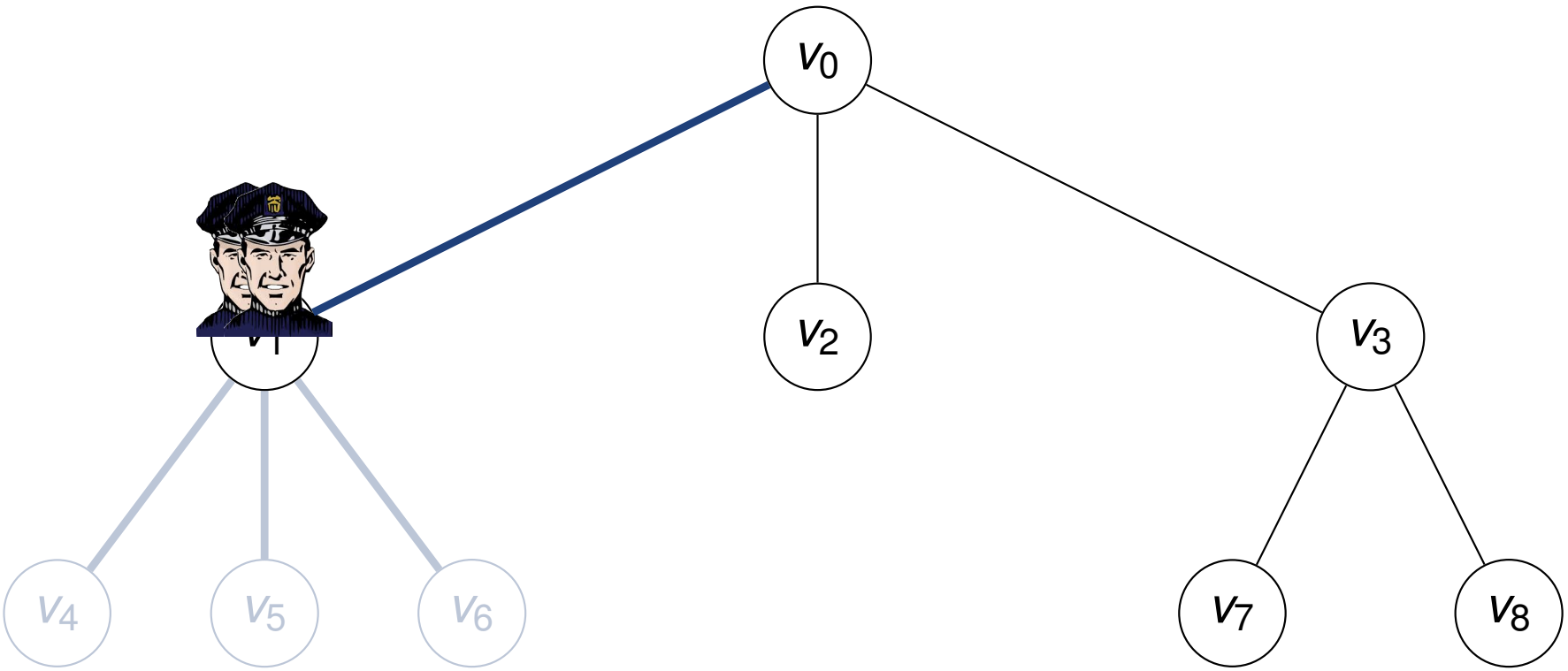

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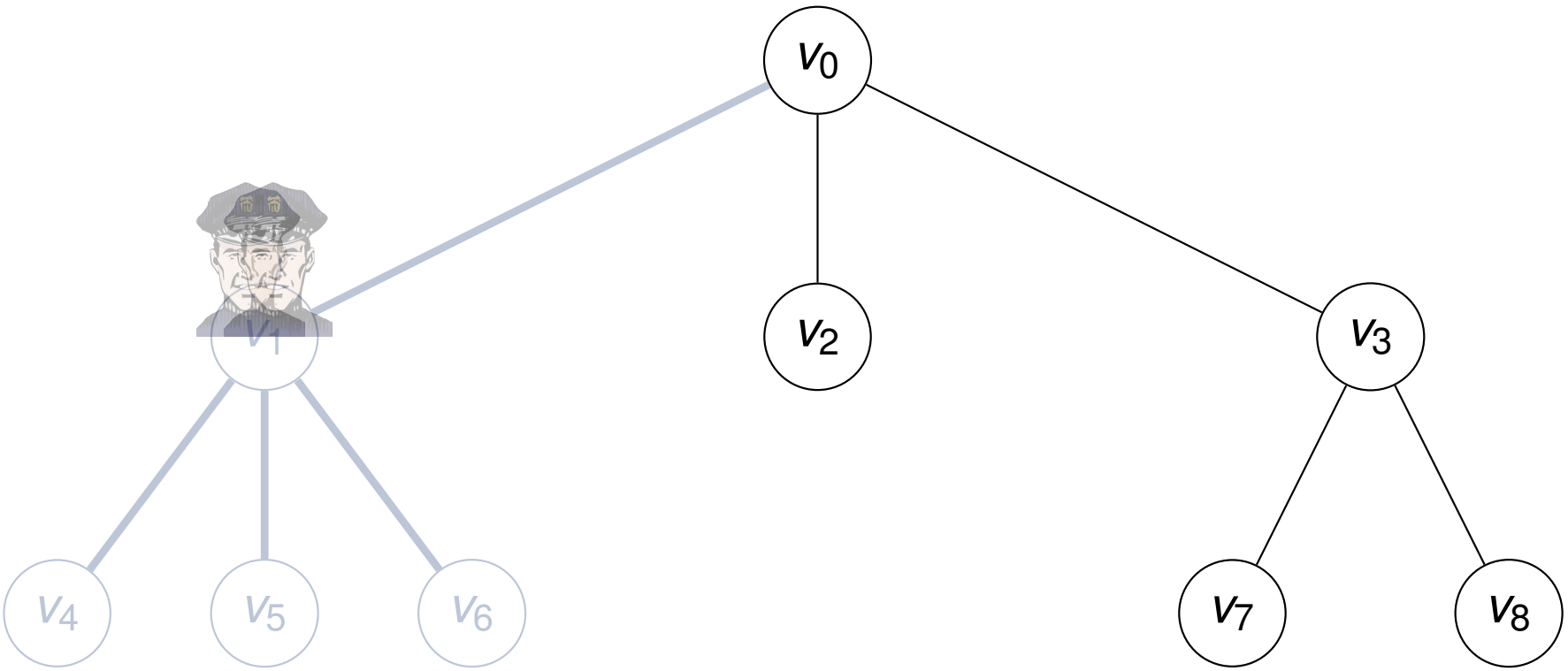

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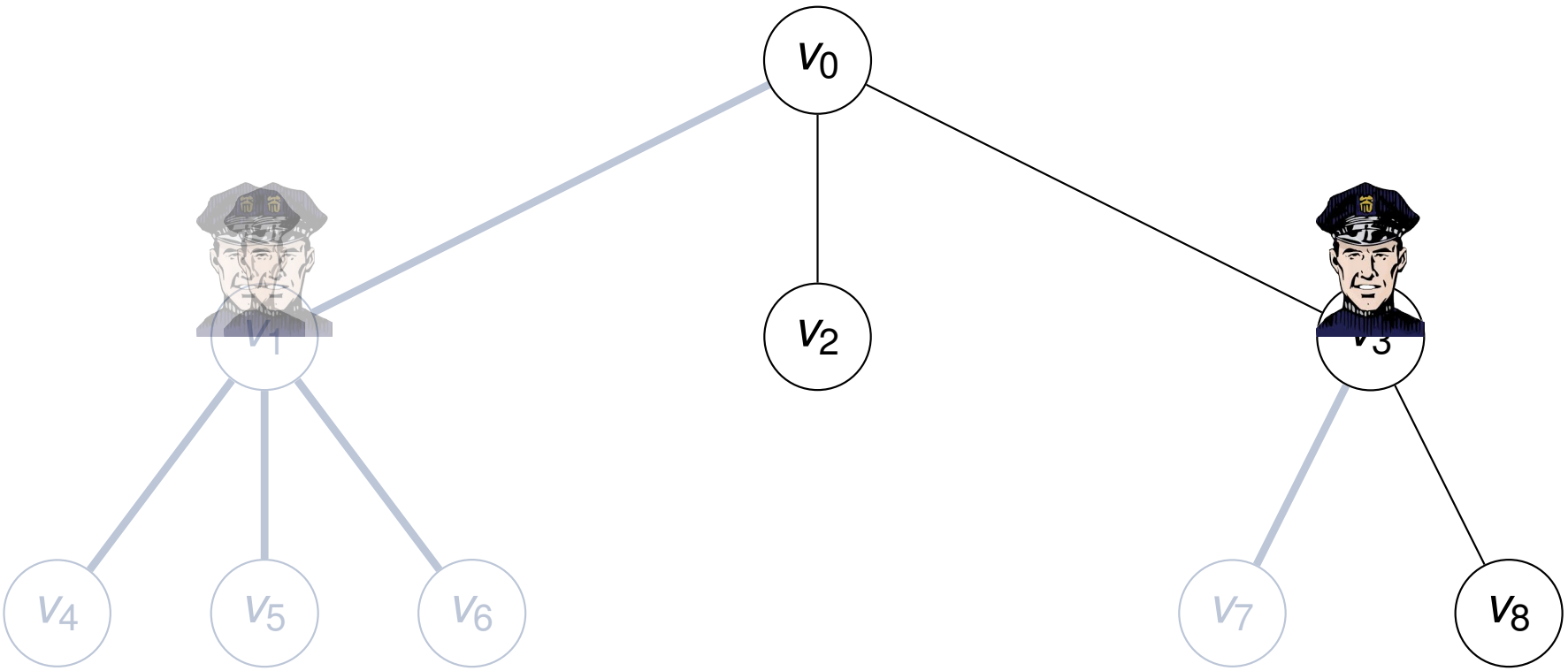

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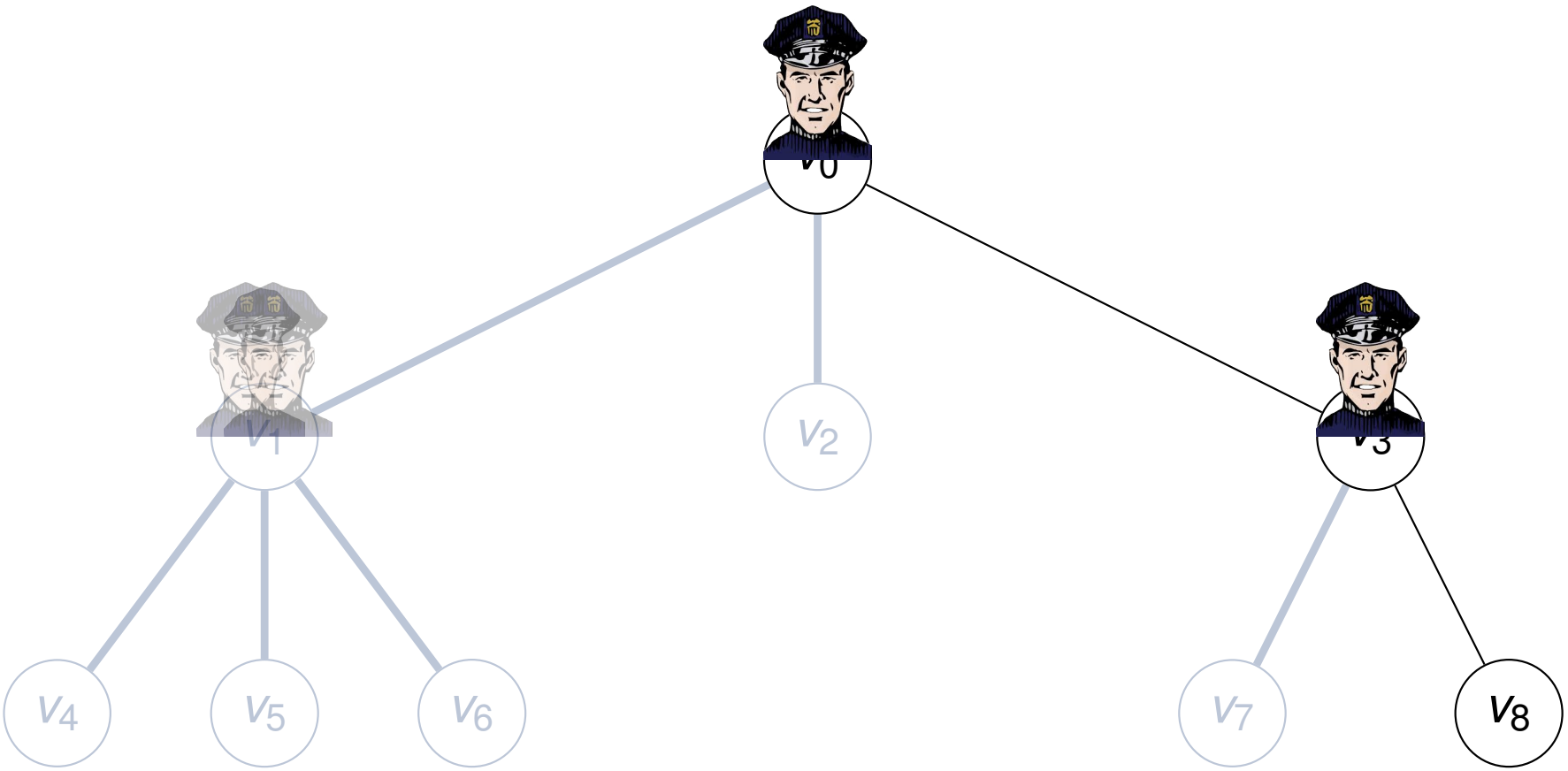

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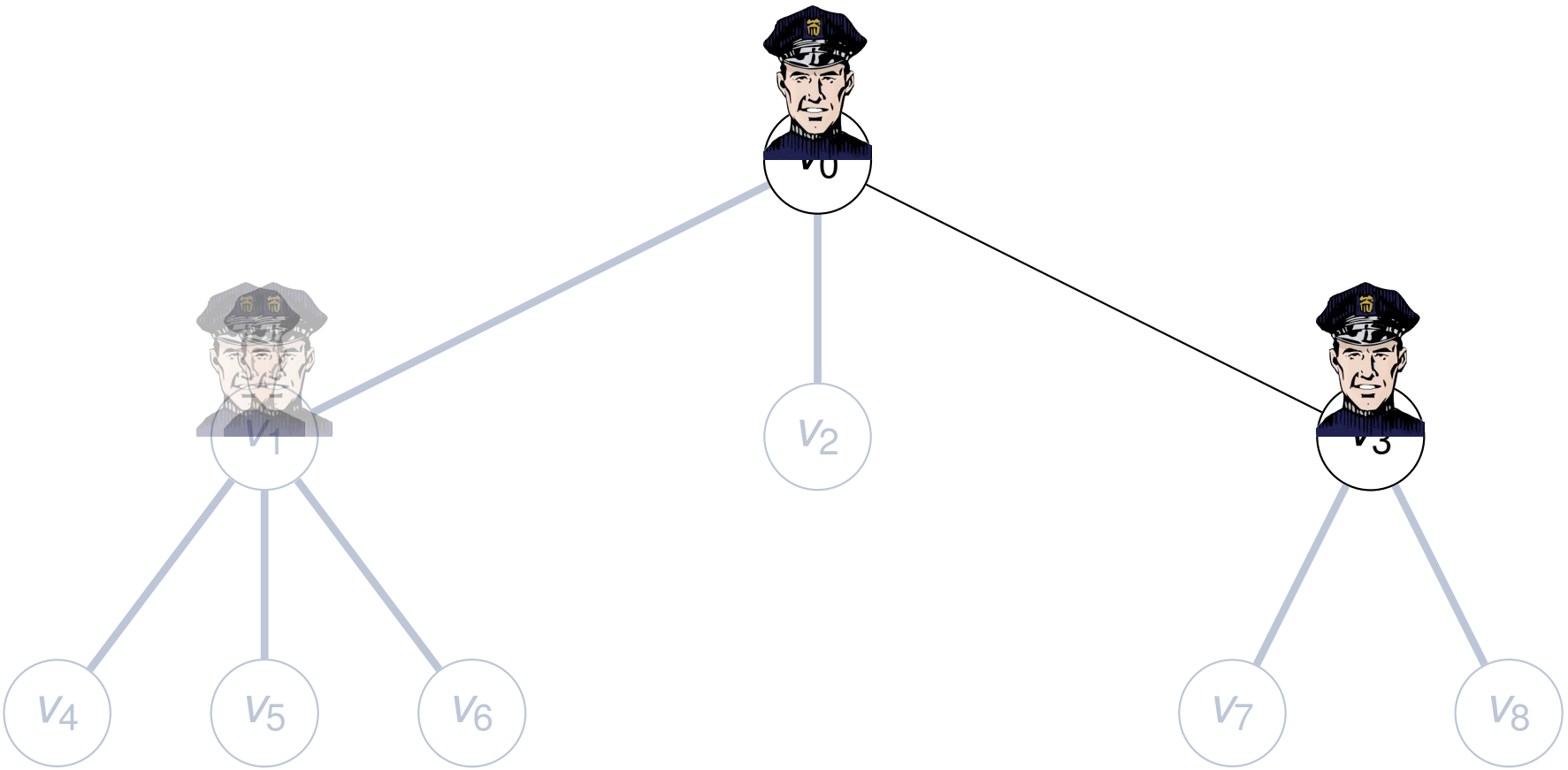

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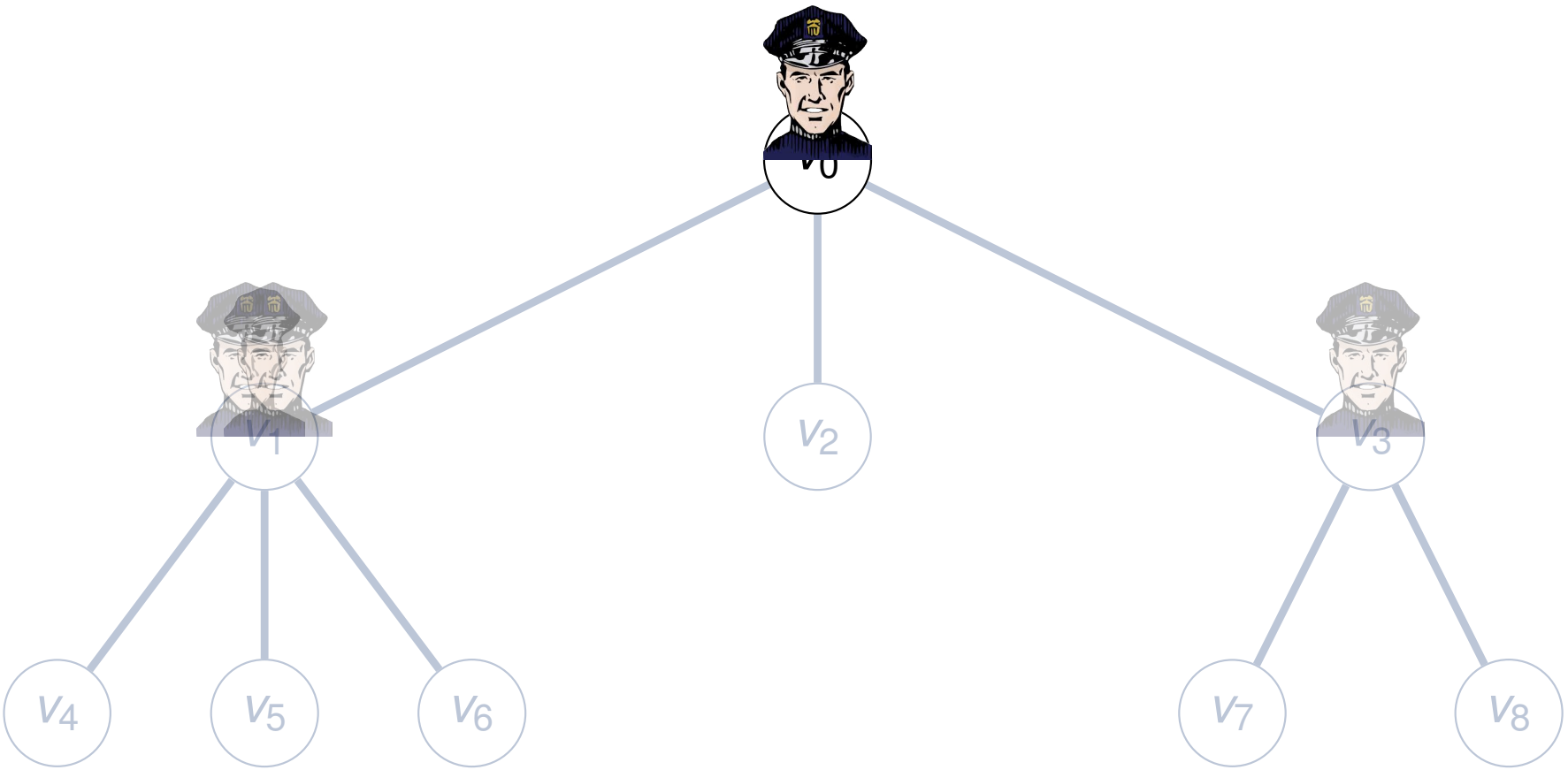

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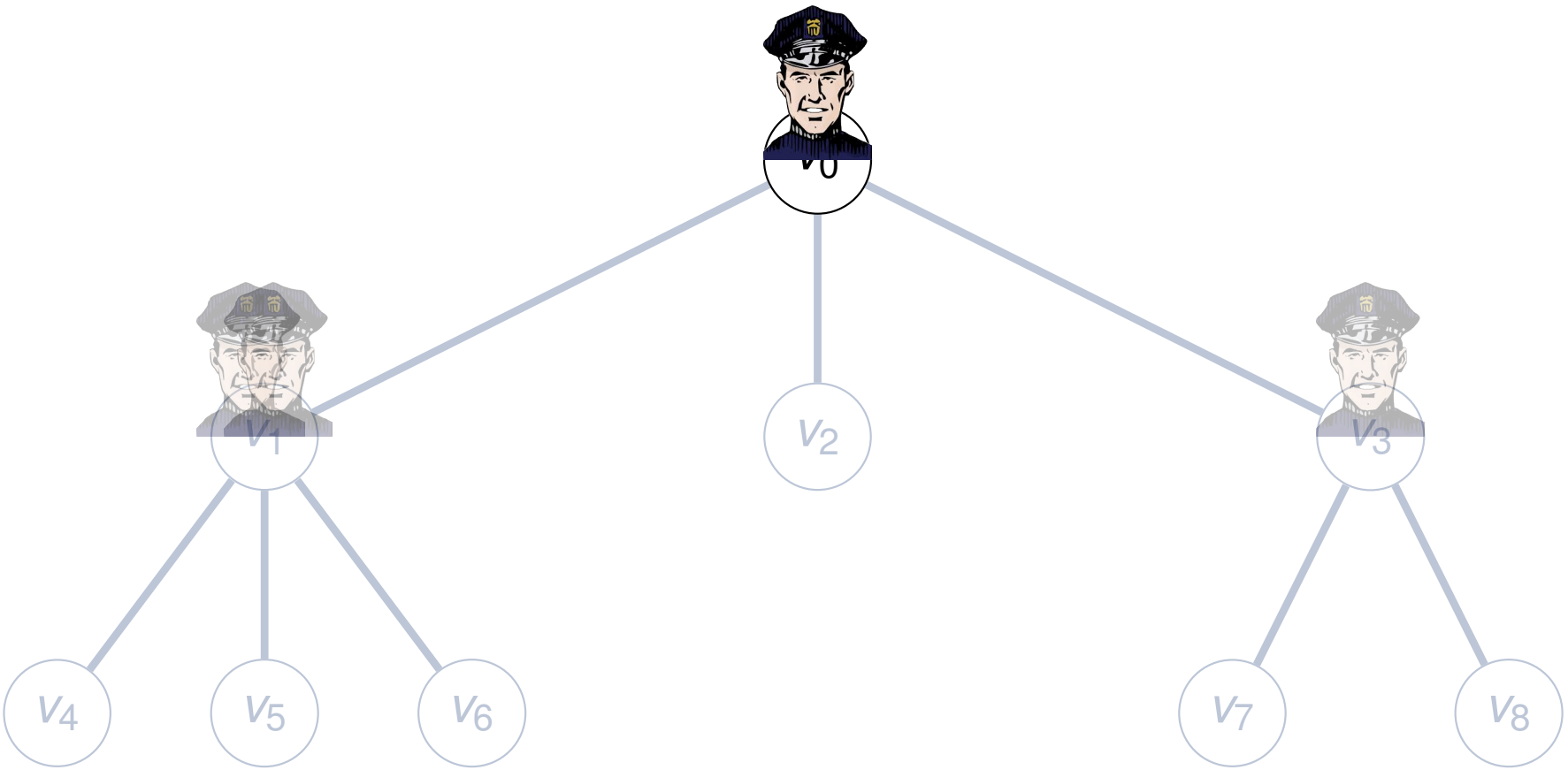

Result: 8 edges, 4 detectives

Cheese Patrol

When we are done with a subtree

Cheese Patrol

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- We used an optimal number of detectives

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- If subtree has even number of edges, half as many detectives

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⇨ Used to watch edge to parent

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- If odd number of edges, one det. has capacity for one edge

⇨ Used to watch edge to parent

- Unless parent is root

Cheese Patrol

Now assume graph is arbitrary

Cheese Patrol

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Somewhat the same problem

Cheese Patrol

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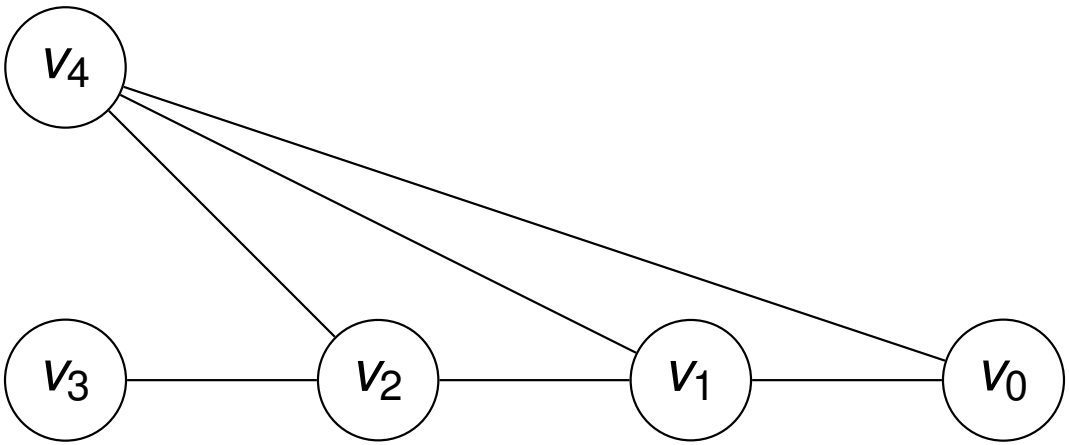
Somewhat the same problem



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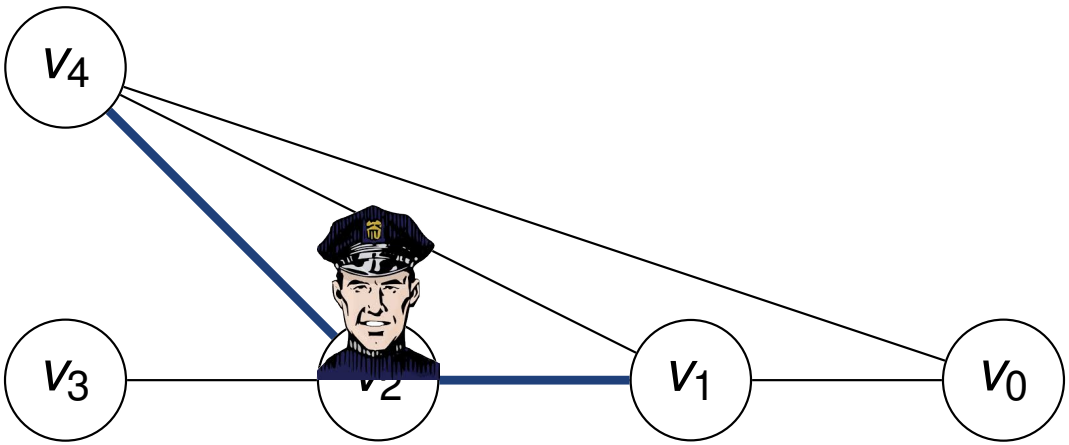

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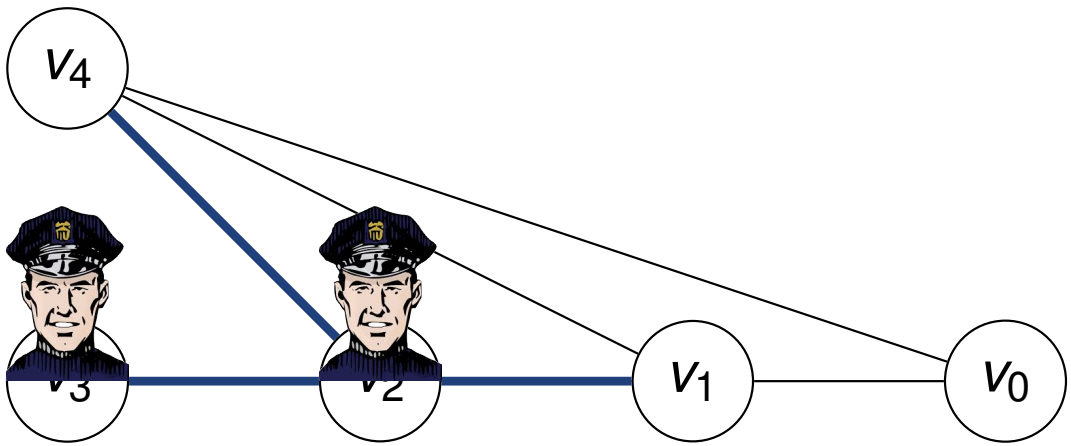

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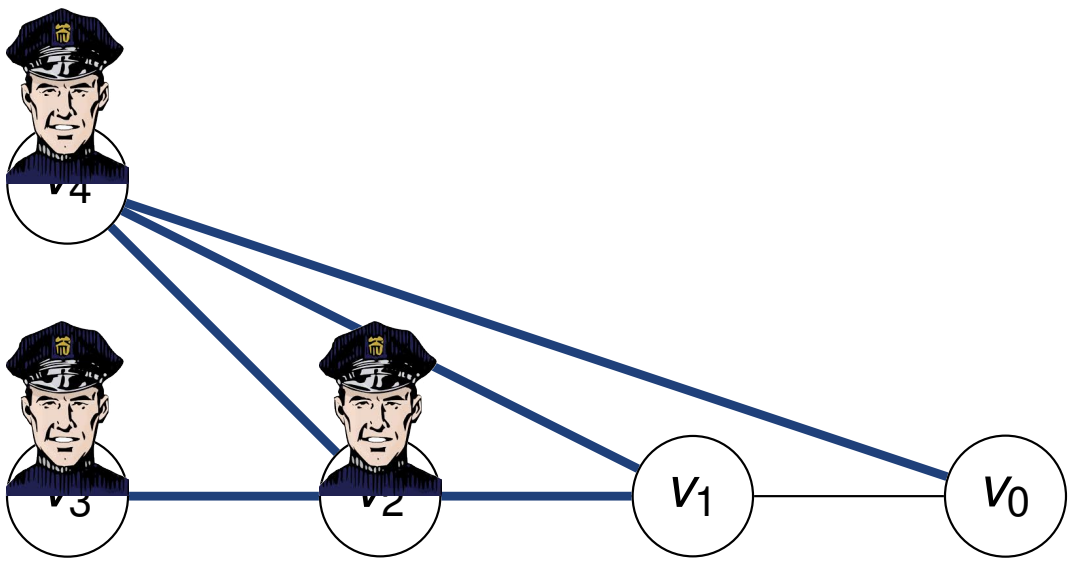

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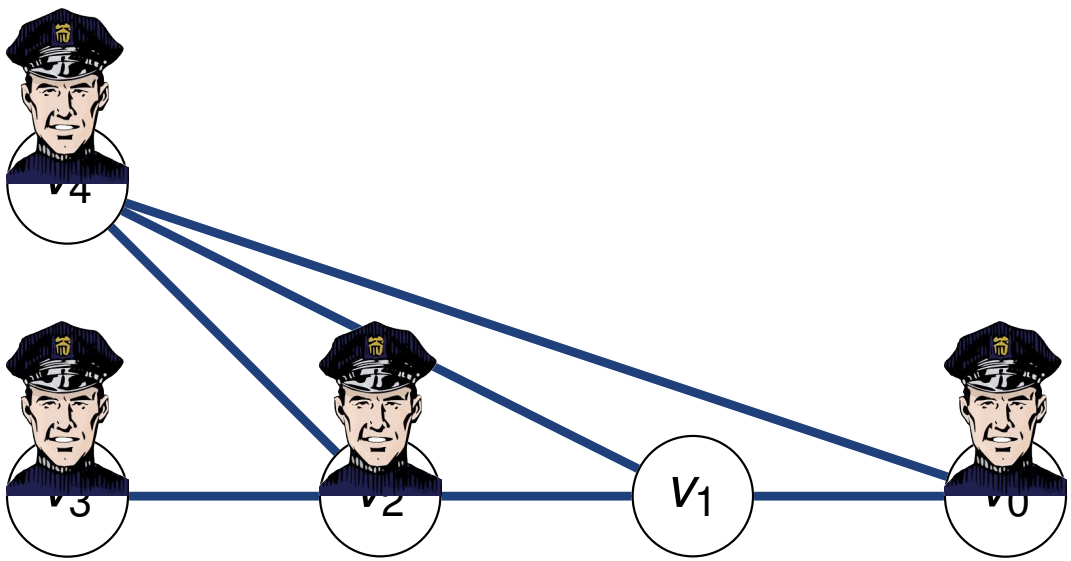

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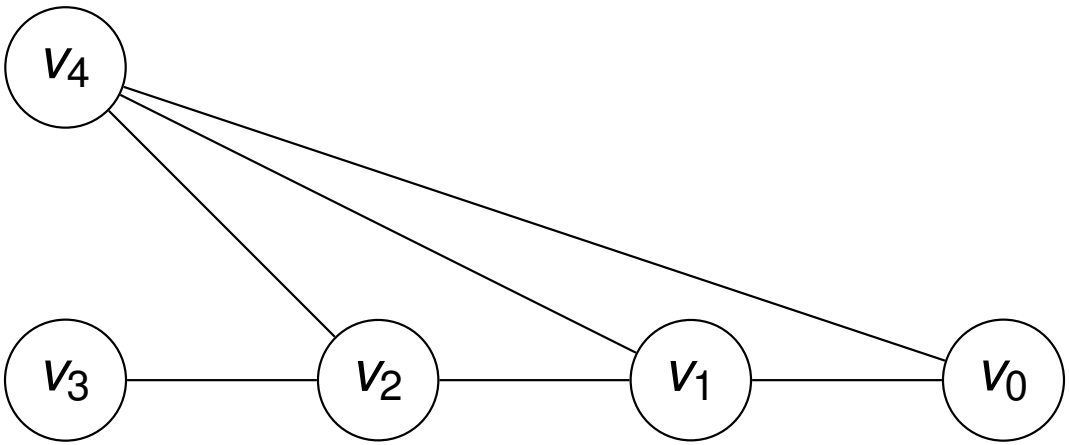
Now assume graph is arbitrary

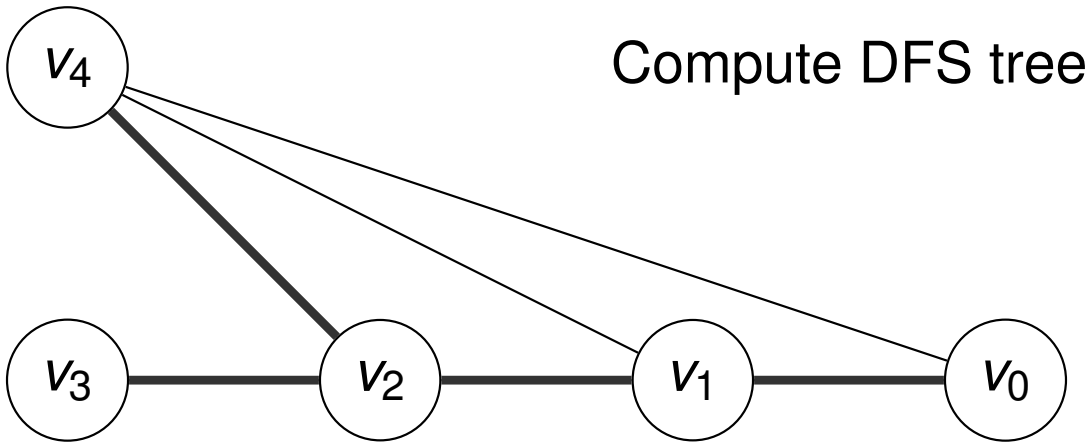
Somewhat the same problem



Cheese Patrol

Now assume graph is arbitrary

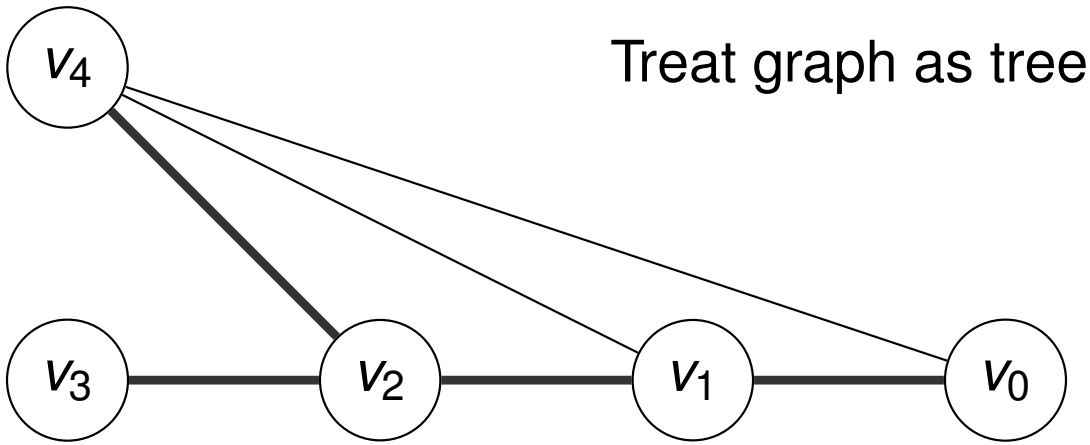
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

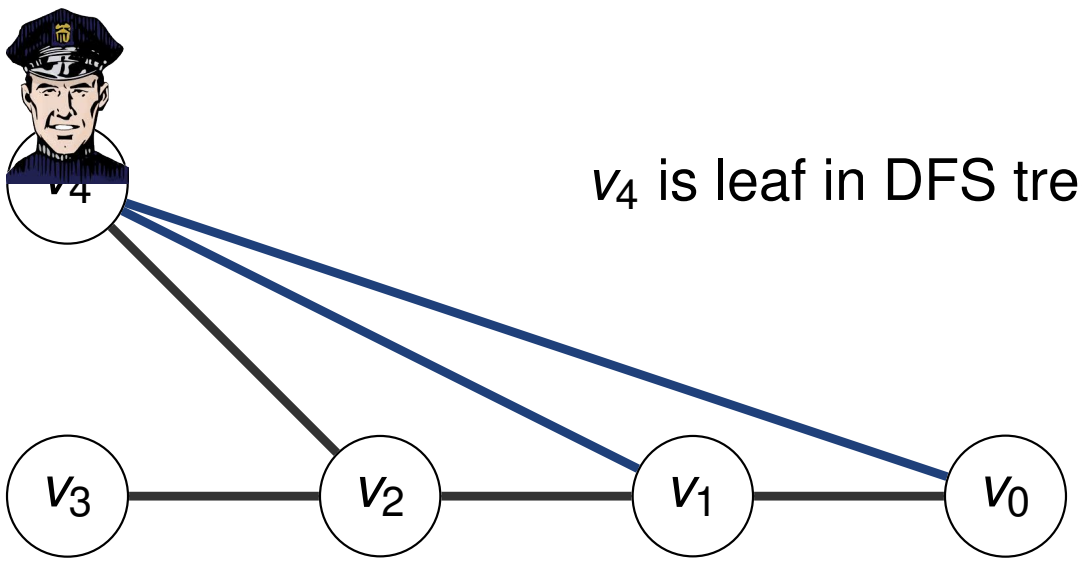
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

Use DFS to avoid this problem

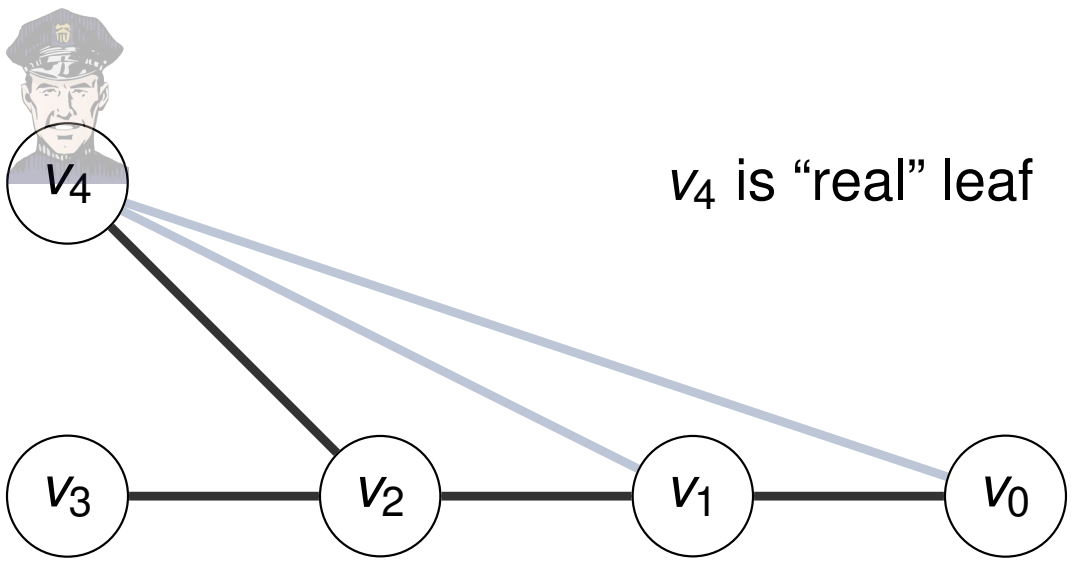


v_4 is leaf in DFS tree

Cheese Patrol

Now assume graph is arbitrary

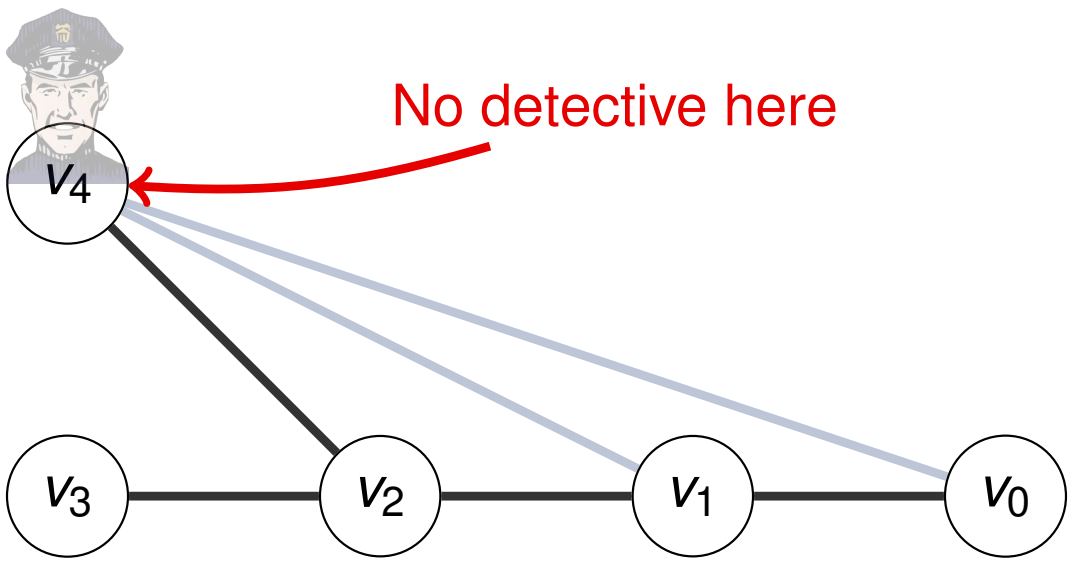
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

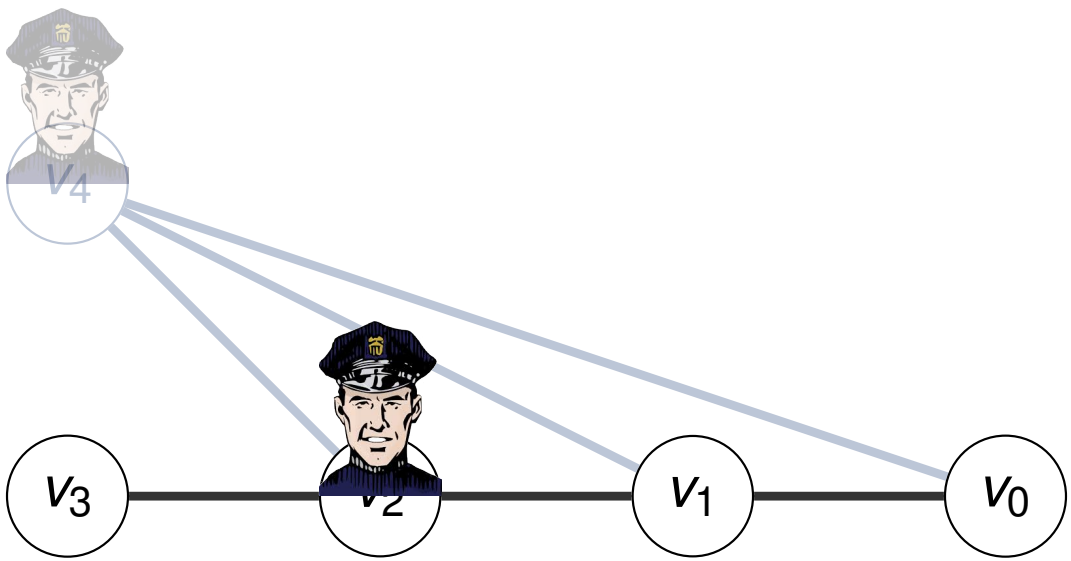
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

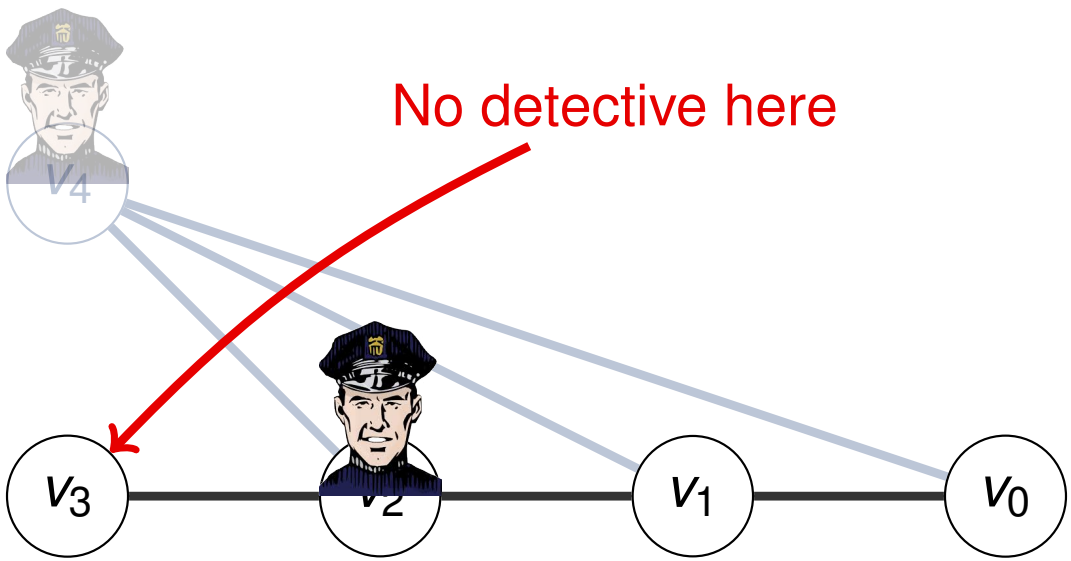
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

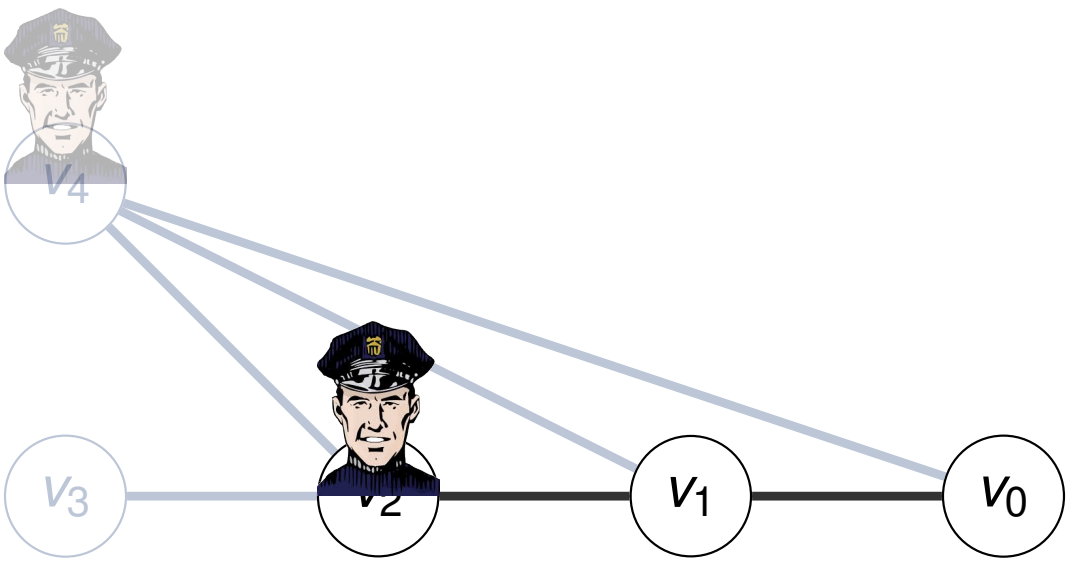
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

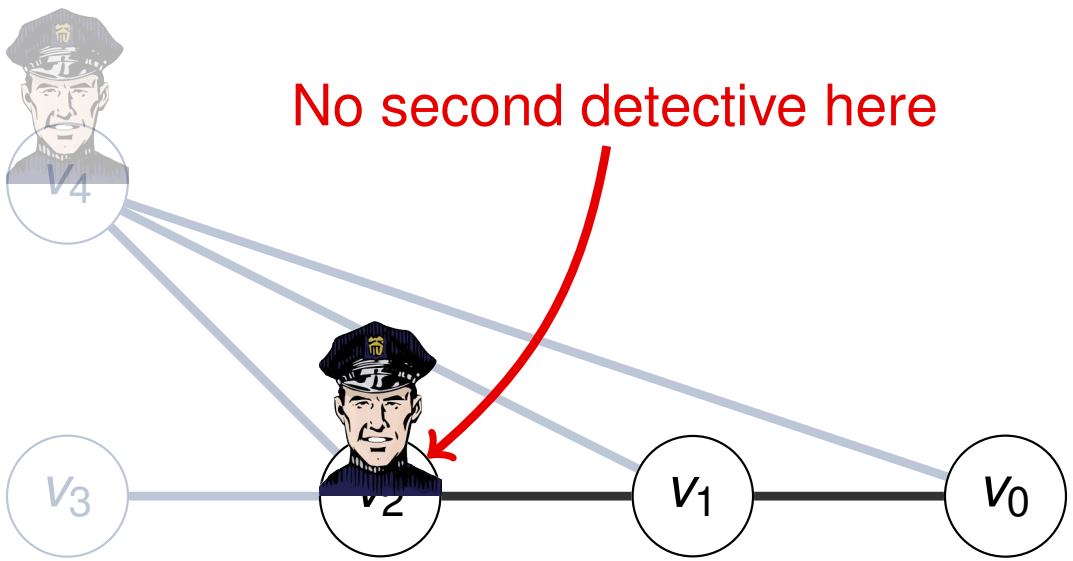
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

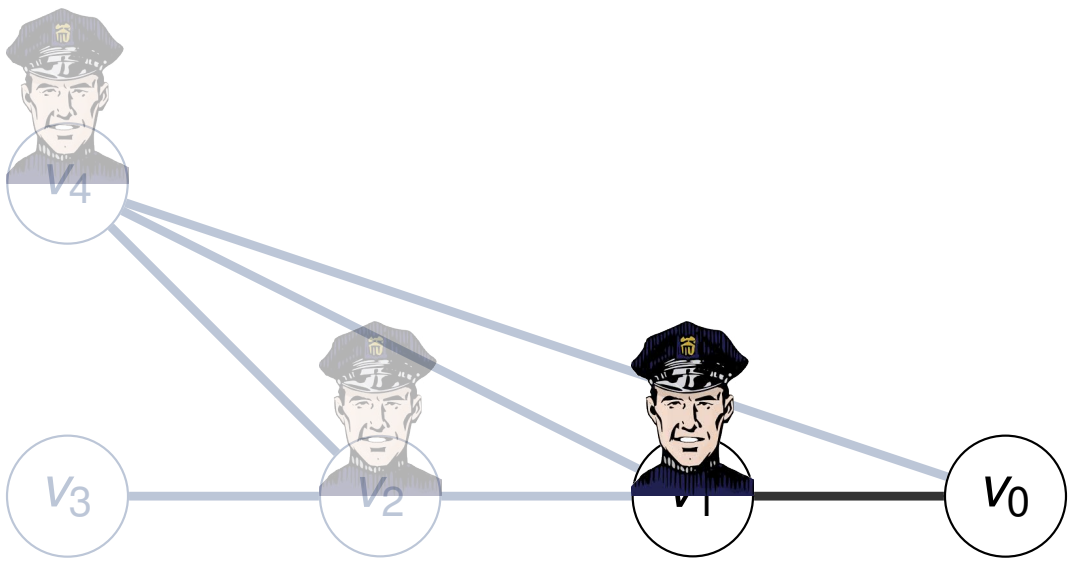
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

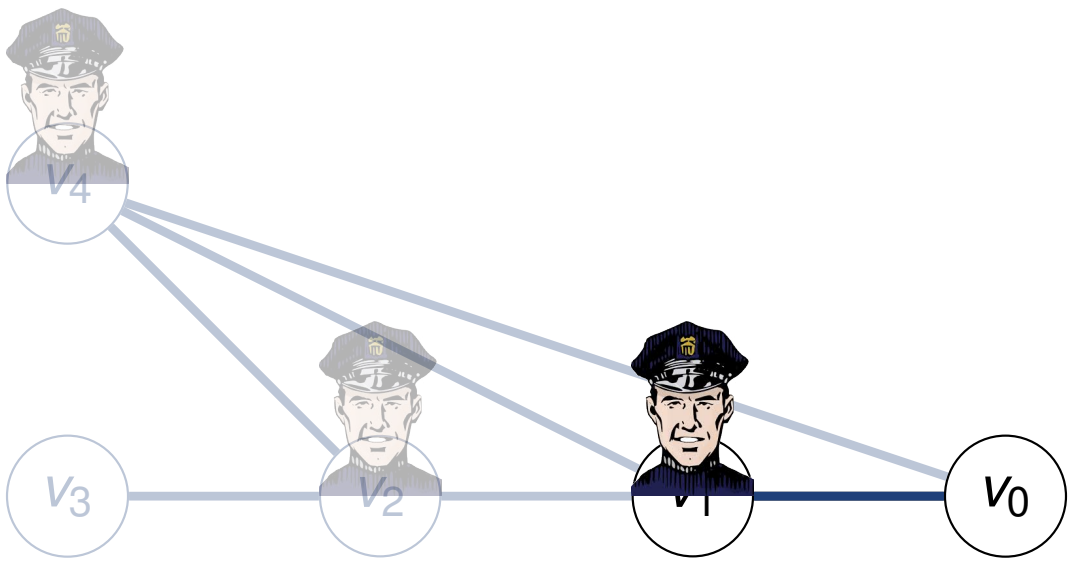
Use DFS to avoid this problem



Cheese Patrol

Now assume graph is arbitrary

Use DFS to avoid this problem



Thanks for the attention