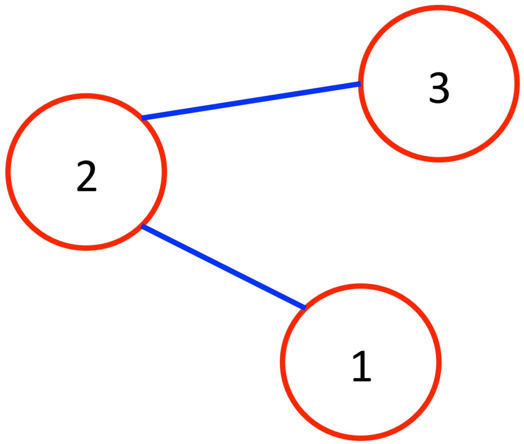
Graphs

TYPES AND PROPERTIES

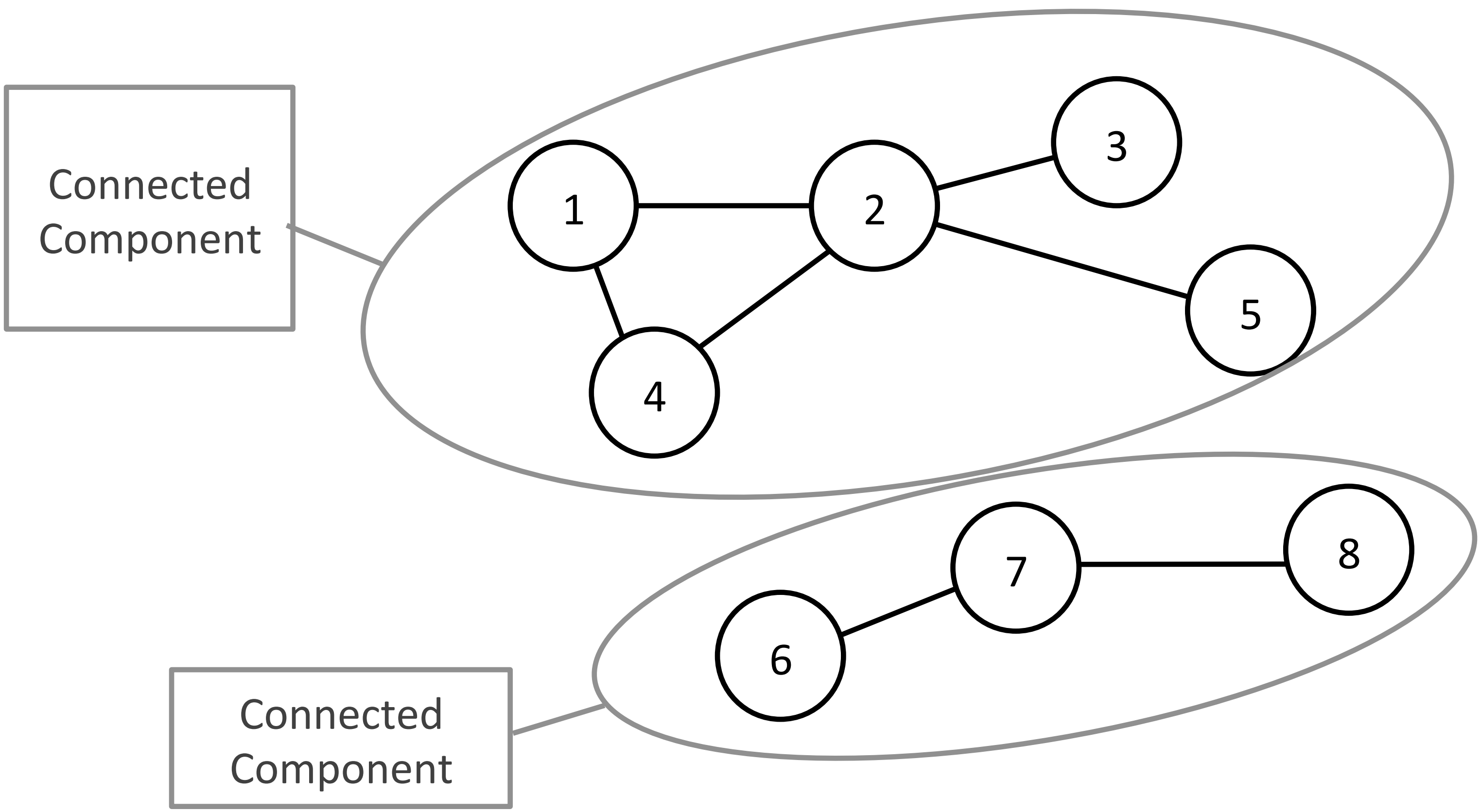
Mathematical description

- Tuple of two sets: G = (V, E)
  - Vertices/Nodes
  - Edges
  - Number of Edges: |E|
  - Number of Nodes: |V|
- Adjacent Nodes: Neighbours



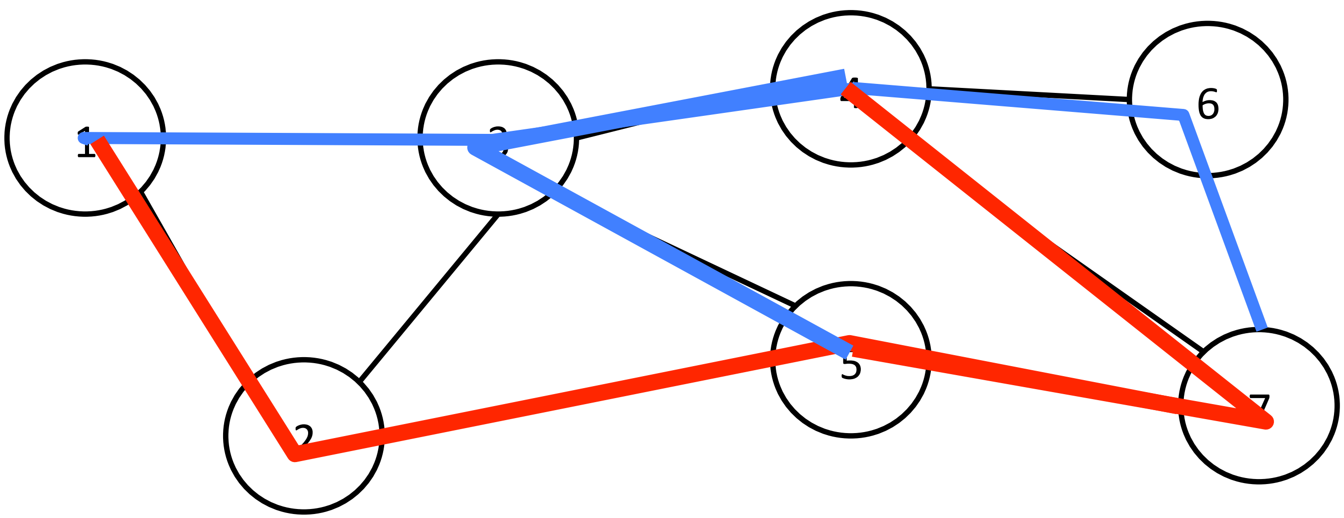
Connected Components

- No connected Graph:

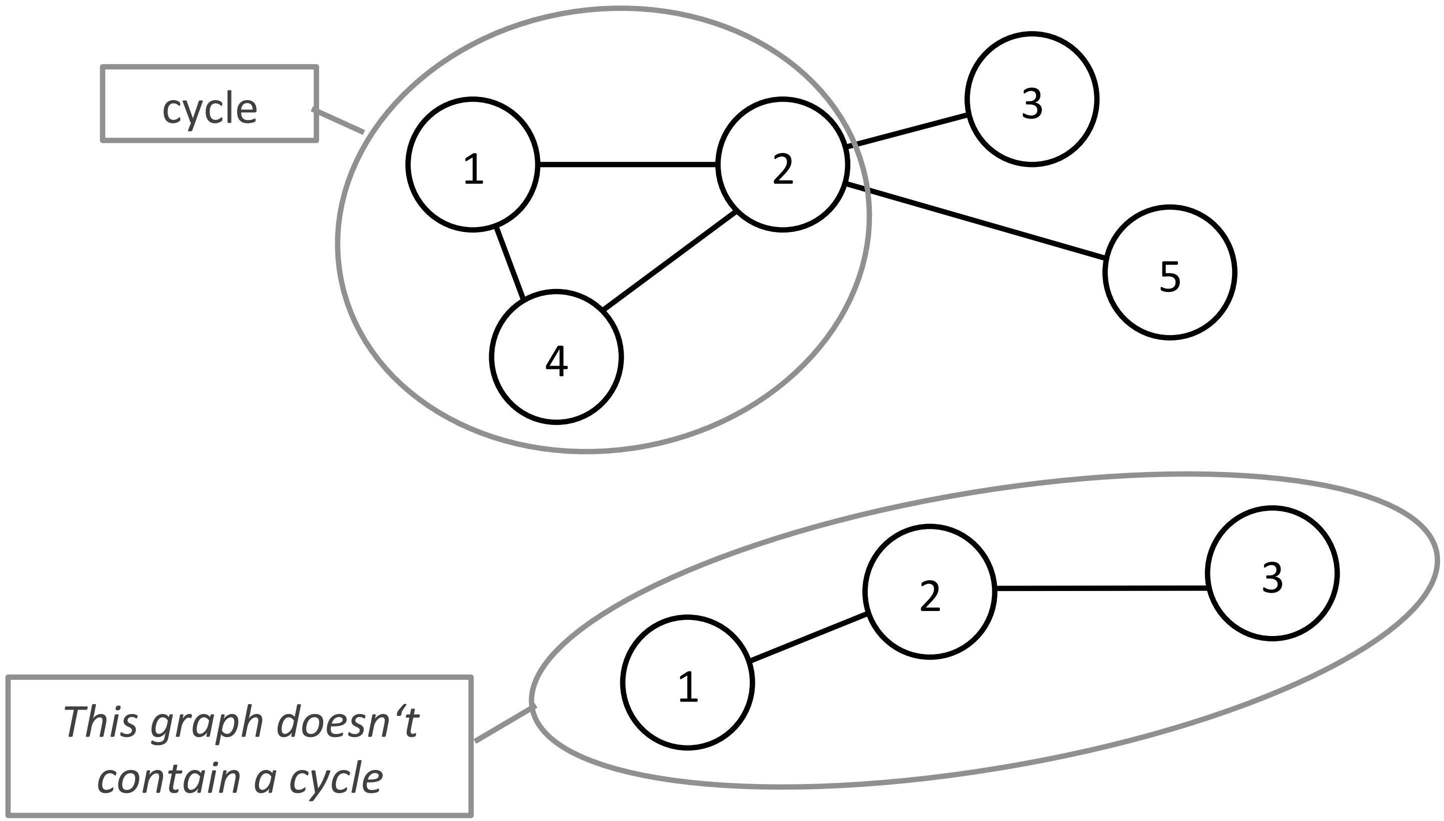


k-connectivity

- One can pick any k-1 Nodes, remove them and the graph stays connected.
- **Mengers Theorem:** this is equivalent to that for any two Nodes there are at least k paths connecting them that share only the start and end node.

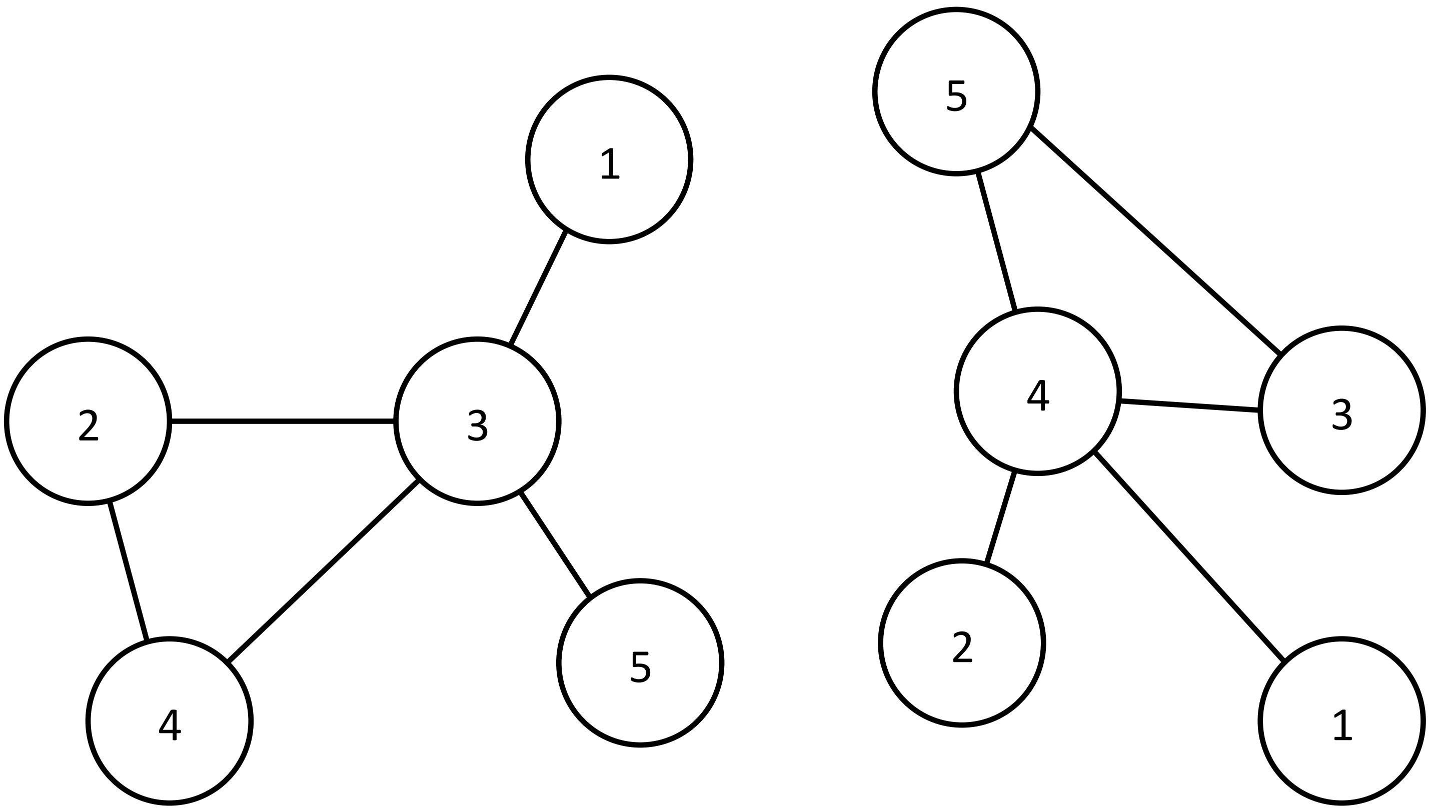


Cycles



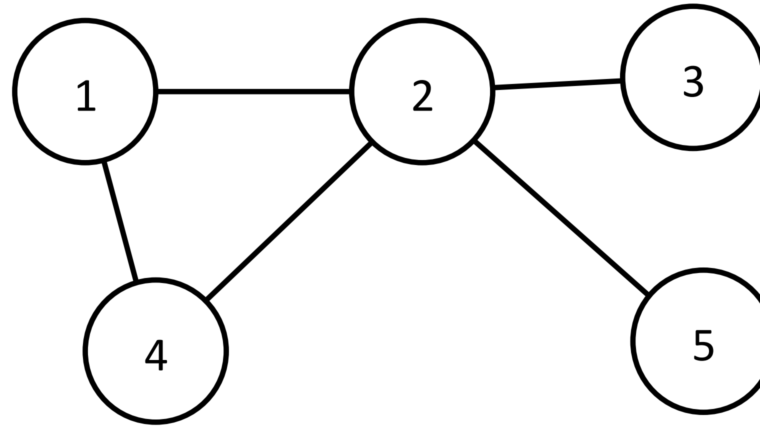
Isomorphism

= equal structure

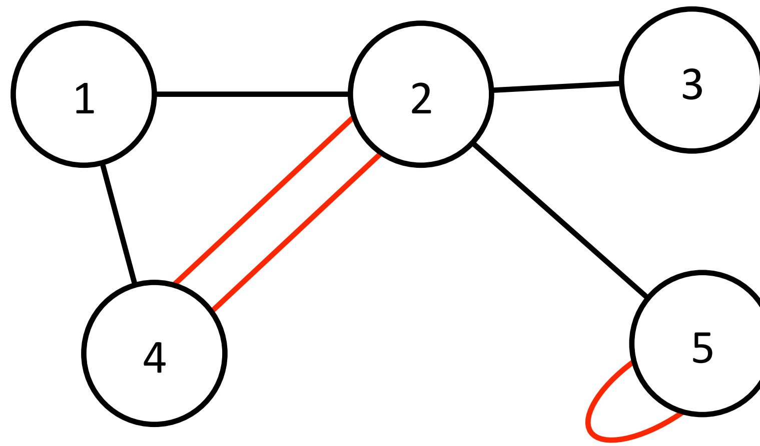


Types

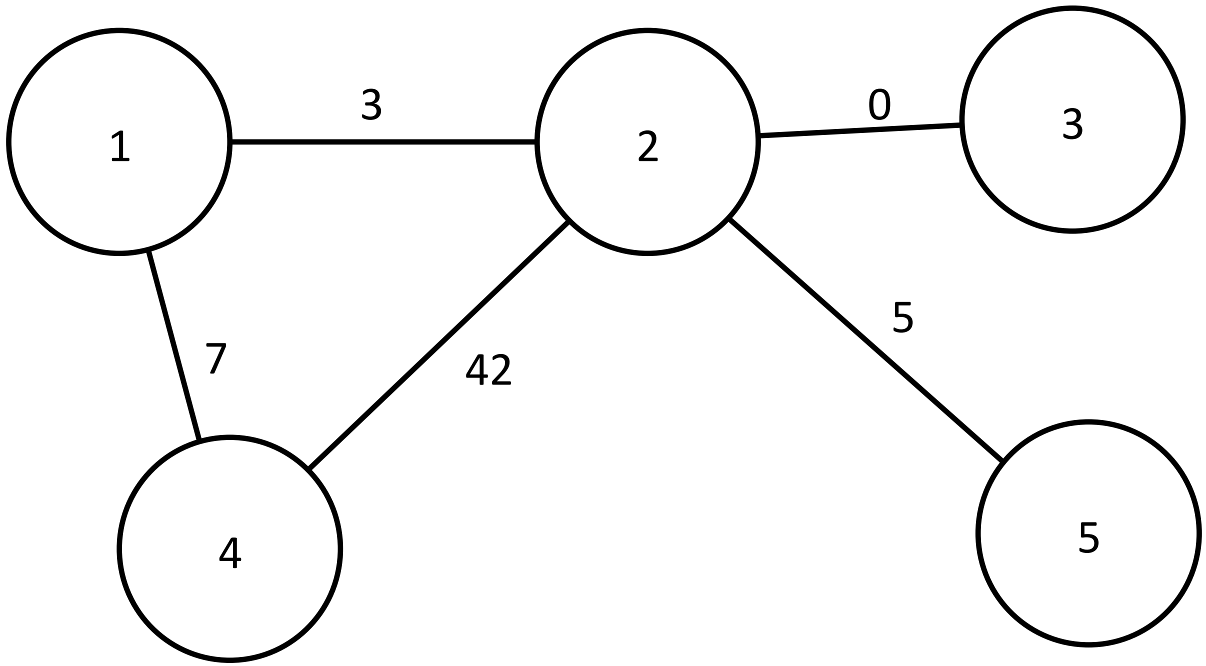
- simple graph:



- multigraph:

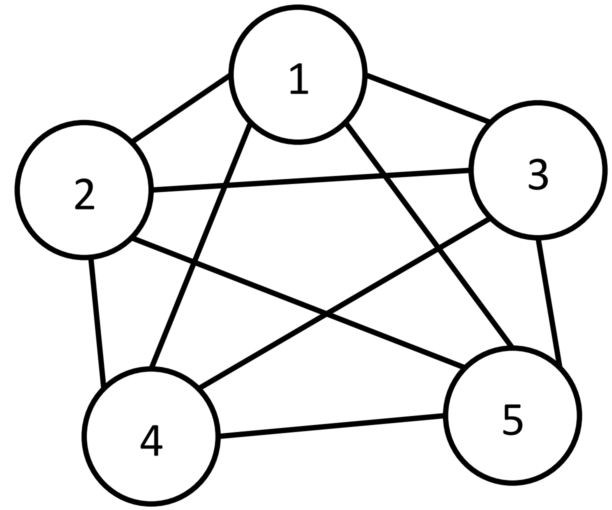


Weighted graph

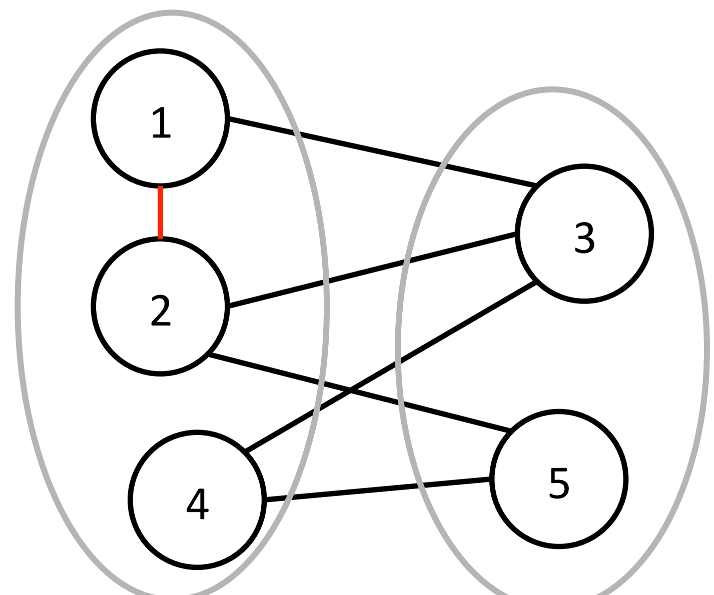


Special structures

- Complete graph: Every node is connected to every other node

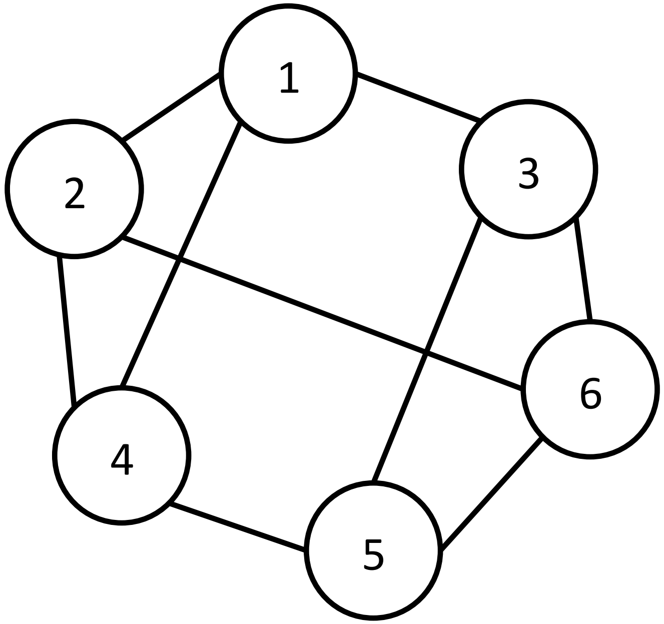


- Bipartite Graph: The nodes can be divided into **two groups** such that no edge connects nodes from the same group:

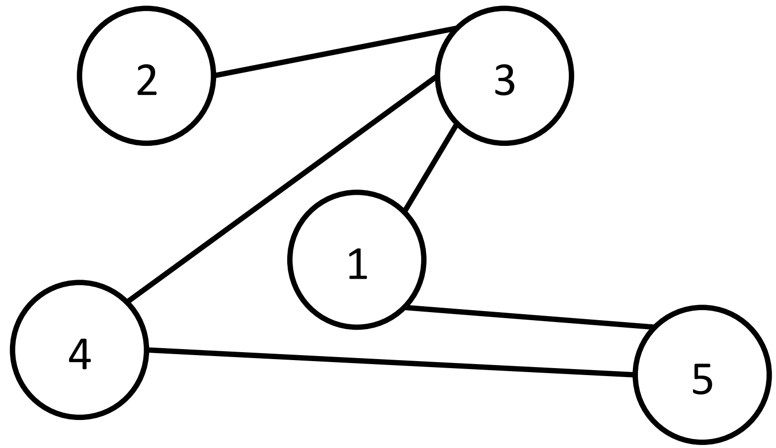


Degree of a vertex

- deg(v) = # edges connected to v
- *k-regular* graph: Every node has degree *k*



Repetition: Multiple Choice



	A	B	C	D	E
contains a cycle	no	yes	no	yes	yes
k-connected	2	2	2	1	1
multi-/single graph	single	single	multi	single	single
weighted	no	no	yes	no	no
complete	yes	no	yes	no	no
bipartite	yes	yes	no	yes	no
k-regular	yes, 2	no	yes, 1	no	no

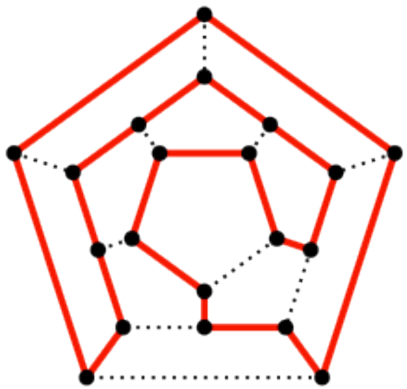
THEOREMS ABOUT GRAPHS

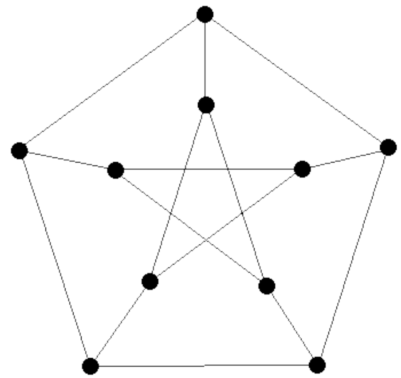
Theorems

- A simple graph has O(V^2) edges
  - precisely: At most |V| (|V|-1)/2
- A simple Graph that has |V| or more edges always has a cycle.
- The sum of the degrees of all nodes is 2|E|
- The number of nodes with odd degree is even
- The average of the degrees of all nodes is 2|E|/|V|

Hamilton cycle

- It is a cycle in a graph that traverses all nodes



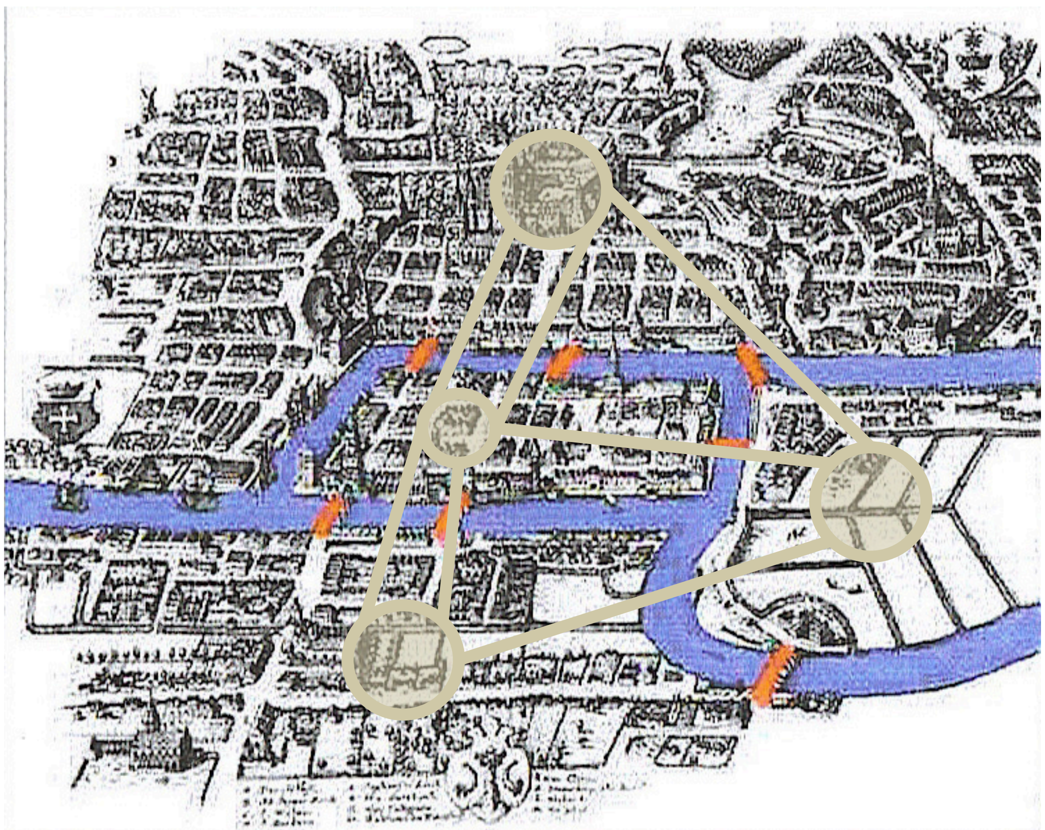


Petersen graph

- It is NP-Hard to find out if a graph contains a hamilton cycle

Eulerian cycle

- It is a cycle that traverses all edges



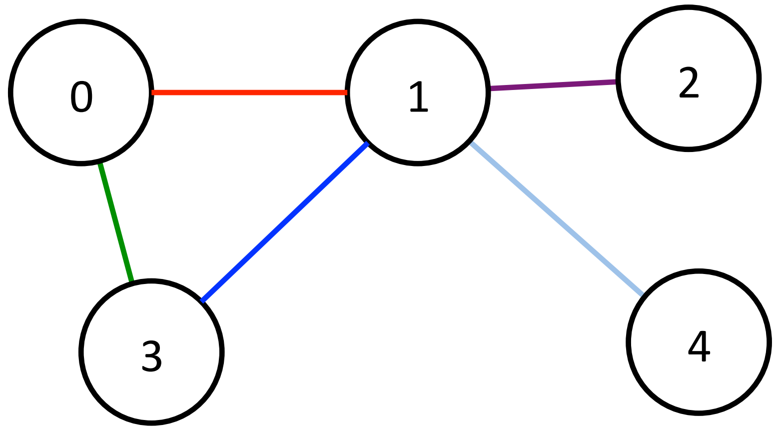
Eulerian cycle

- It is a cycle that traverses all edges
- It only exists if the degree of all all nodes is even
- An eulerian path exists if there are exactly two nodes with odd degree

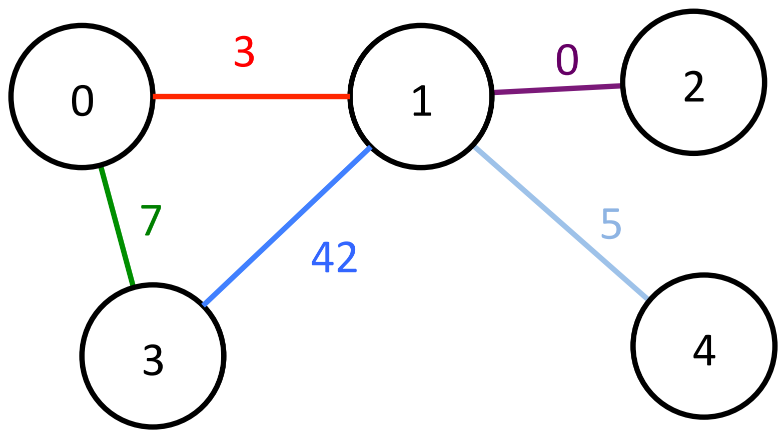
IMPLEMENTIERUNGEN

Adjazenzmatrix

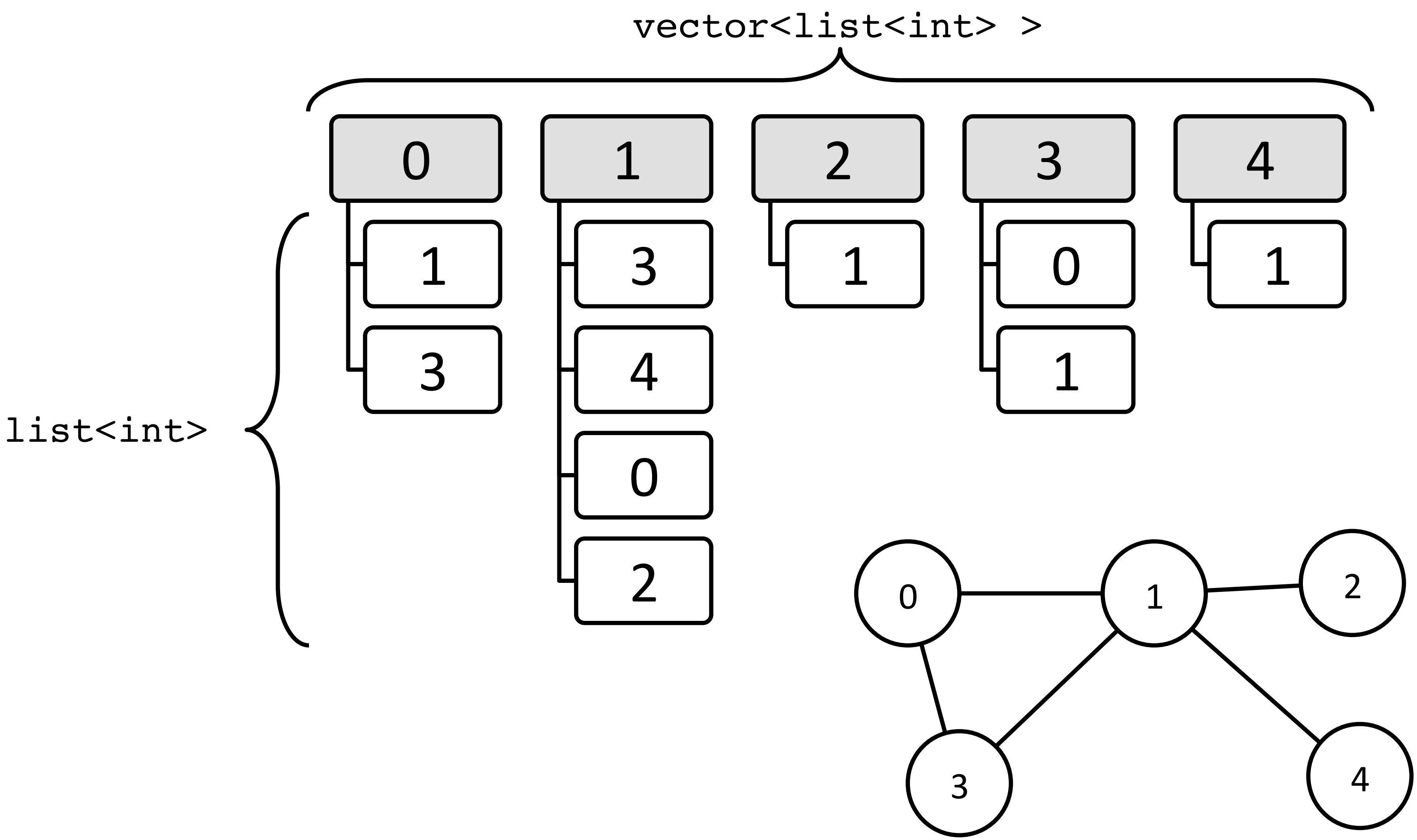
	0	**1**	**2**	**3**	**4**
0	0	1	0	1	0
1	1	0	1	1	1
2	0	1	0	0	0
3	1	1	0	0	0
4	0	1	0	0	0



	0	**1**	**2**	**3**	**4**
0	-1	3	-1	7	-1
1	3	-1	0	42	5
2	-1	0	-1	-1	-1
3	7	42	-1	-1	-1
4	-1	5	-1	-1	-1



Adjazenzliste

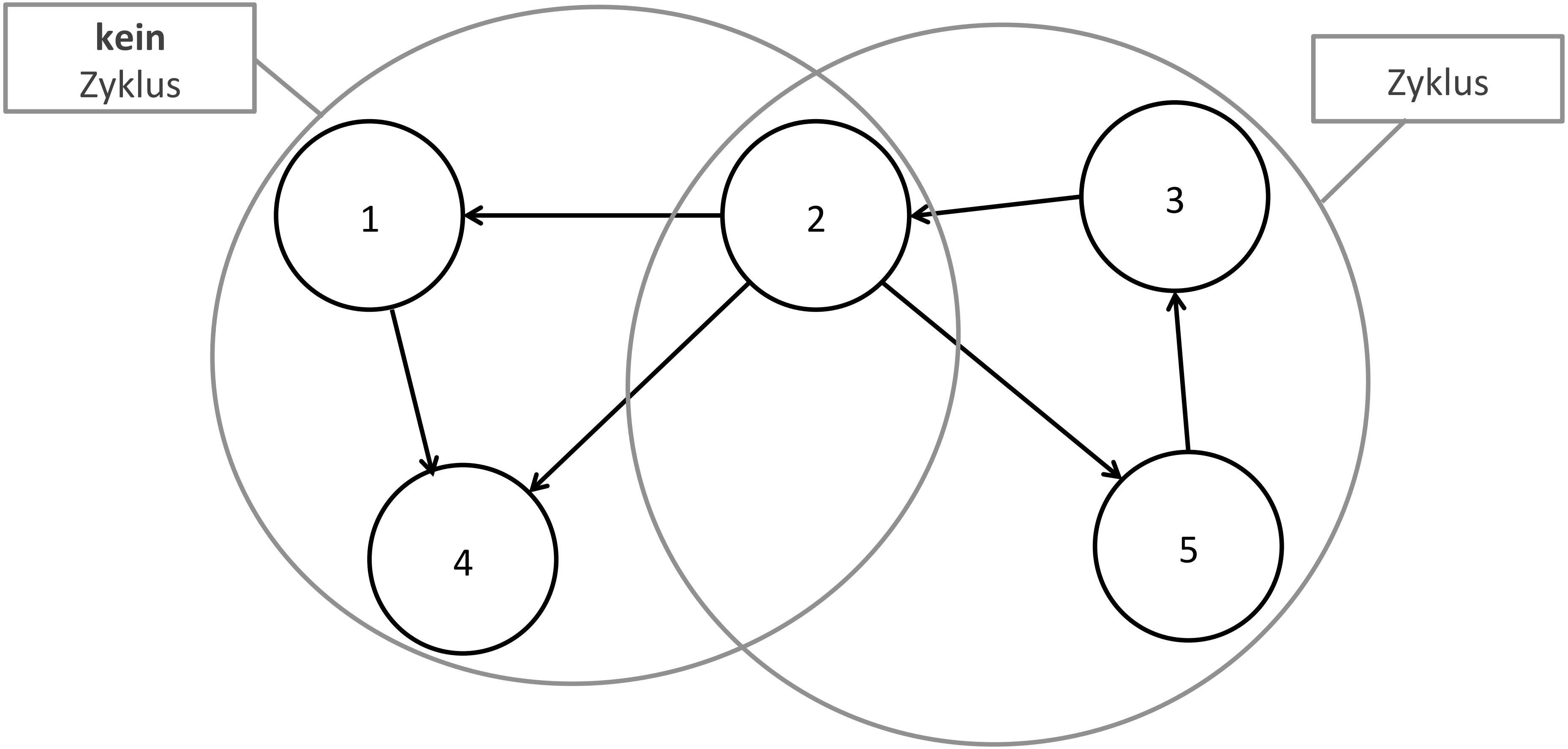


Vergleich Adjazenzliste/matrix

	Adjazenzliste	**Adjazenzmatrix**
Speicherplatz	V+E	V^2
Kante einfügen	1	1
Kante löschen	E	1
Existiert Kante?	E	1
Finde alle Kanten eines Knoten	E	V
Traversieren	V+E	V^2

DIGRAPHEN & TOPOSORT

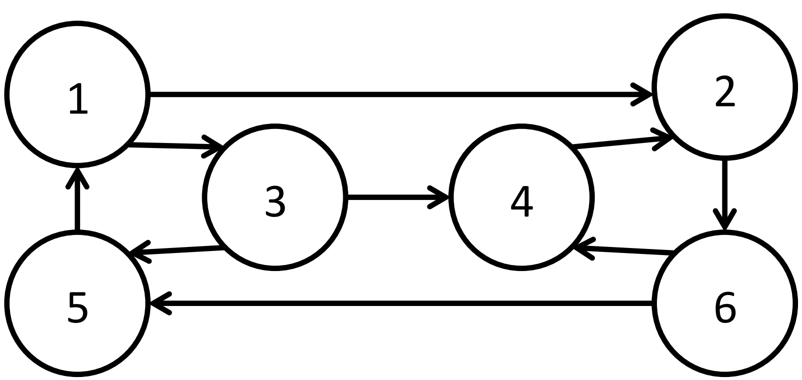
Directed Graph

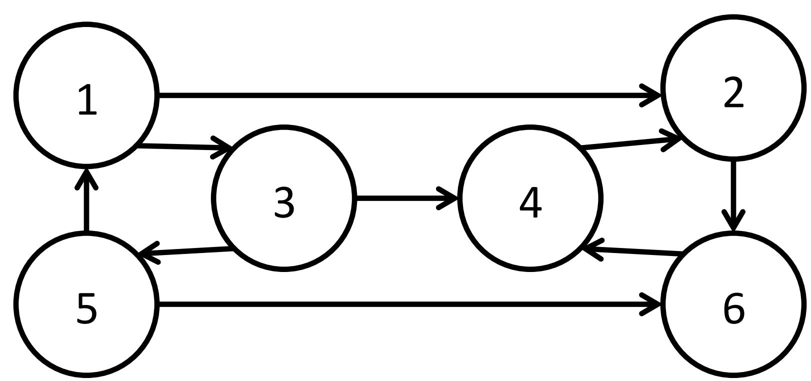


- DAG = acyclic directed graph

Strong and weak connectivity

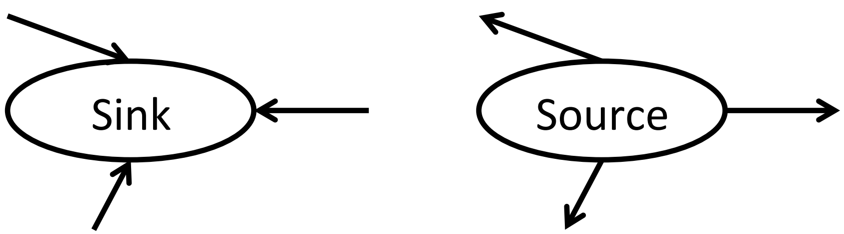
- strongly connected if there is a directed path between any two vertices
- weakly connected if the corresponding undirected graph is connected





Degree, Source and sink

- Indegree and Outdegree of a node
- Source: Indegree = 0, Sink: Outdegree = 0



- Every DAG has a source and a sink.

Schedueling