

Binary search problems

Dániel Korándi

Department of Mathematics, ETH Zürich

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The smallest feasible solution

Problem

You are given an array $a[1..n]$ of 0's and 1's, and you know that the first few values in a are all 0's, and the remaining are all 1's. Give an algorithm that finds the smallest index p such that $a[p] = 1$ in time $O(\log n)$. In other words, find the p satisfying $a[1] = a[2] = \dots = a[p-1] = 0$ and $a[p] = a[p+1] = \dots = a[n] = 1$.

- $i := 1; j := n$
- **while** $i < j$ **do**
 - $m := \lfloor (i+j)/2 \rfloor$
 - **if** $a[m] = 1$ **then** $j := m$
 else $i := m + 1$
- **return** i

Finding a fixpoint

Problem

You are given an array $a[1..n]$ of n distinct (and not necessarily positive) integers. Assume that the entries of a are sorted so that $a[1] < a[2] < \dots < a[n]$. Give an algorithm that outputs an i satisfying $a[i] = i$ if such an index exists, and its running time is $O(\log n)$.

Problem

What if all the entries are positive? Can you do better?

Homework

You are given a sorted array of n different numbers from the set $\{1, \dots, m\}$, where $n < m$. Give an algorithm that finds a number p between 1 and m missing from the array and that runs in time $O(\log n)$.

Finding the peak

Problem

We know that the n entries in the array a are strictly increasing until some index p and they are strictly decreasing afterwards:

$$a[1] < \dots < a[p-1] < a[p] > a[p+1] > \dots > a[n].$$

Give an algorithm that finds the index p in $O(\log n)$ steps.

Homework

You are given an *unsorted* array $a[1..n]$ of n different numbers. Give an algorithm that finds a local maximum of a in time $O(\log n)$. ($a[i]$ is a local maximum if it is greater than both its neighbors $a[i-1]$ and $a[i+1]$ or if $i=1$ or n , then its one neighbor.)

Problem

You have n containers in a row, and you need to load them onto k trucks in such a way, that each truck carries a set of consecutive containers. You are given the weights of the containers, and you want to distribute them among the trucks as evenly as possible. This means that you want the maximum total weight loaded to a truck to be as small as possible. Give an algorithm that determines this minimum.

Homework

Give a dynamic programming solution that runs in $O(nk)$ time.

The widest path problem

Problem

Our computer network has a graph structure with n computers and m data links connecting some pairs of computers. We know the bandwidth of each link (the speed of the connection). We want to send data from computer A to computer B . Of course the speed of the transmission through a series of links is the minimum bandwidth appearing in this path. Give an algorithm that computes the highest possible transmission speed between the computers A and B .

- http://en.wikipedia.org/wiki/Widest_path_problem

Problem

We are given n points on the plane, and we want to cover at least k of them using a quarter disk around the origin. What is the smallest radius, for which such a quarter disk exists?

Homework

You are given a sorted array of n different numbers from the set $\{1, \dots, m\}$, where $n < m$. Give an algorithm that finds a number p between 1 and m missing from the array in time $O(\log n)$.

Homework

You are given an *unsorted* array $a[1..n]$ of n different numbers. Give an algorithm that finds a local maximum of a in time $O(\log n)$.

Homework

Give a dynamic programming solution to the containers problem that runs in $O(nk)$ time. (Don't use binary search!)

Homework

CEOI 2008 - Snake

Thank you!