

Theory Day in Sarnen



Swiss Olympiad in Informatics

13 February 2019



Manual

At the end of the solving phase the tasks will be assigned to the individual group members. (Uniformly at random.)

Evaluation

The presentation should include the following points:

1. Description of the idea and explanation of the algorithm.
2. Proof of why the algorithm is correct.
3. Analysis of asymptotic runtime and memory consumption.

We mainly focus on the runtime and correctness of the algorithm. The quality of the presentation will also be taken into account.

You may show prewritten code, but no other notes. An empty sheet of paper will be available for you during the presentation.

Your presentation should not take longer than 10 minutes.

Technical remarks

It is not stated according to which criteria you should optimize the program. However, the parameter should be clear in all cases (e. g. number n or length of the input string).

You can assume that arithmetic operations with integers can be done in constant time, irrespective of its size. The representation of an integer requires $O(1)$ memory.

Five nights werewolf

Mouse Daniel invented a new game which is called five nights werewolf. Mouse Bibin would technically like to play, but as they are playing during the game jam, he doesn't pay a lot of attention. What he was able to gather is the following:

In the beginning all n mice sit in a row, each of them having a distinct secret number from 1 to n . They are allowed to pick at most one night and in this night move to any place in the row they want (and take their secret number with them). In the end of each night, everyone has to reveal their secret number (now in the changed order) and afterwards, the mice go back to their starting positions.

Task

You are given n and the five sequences of the secret numbers as they were observed after the end of the nights $a_1, \dots, a_n, b_1, \dots, b_n, \dots, e_1, \dots, e_n$. Mouse Bibin has no idea how the game works exactly, but he wants to find out a possible starting arrangement of the secret numbers. Help him do it. (If there are multiple starting positions that are possible, Bibin is happy with any of them).

Example

$n = 3, 123; 123; 123; 123; 231$ (Answer: 1 2 3; if all the mice chose to move in the last night, this might have been the starting position)



Bridges

Contrary to popular belief, Sarnen is a small place in the ocean consisting of several islands. After they realized that it is very unpractical to move between the different islands, mouse Benjamin, the mayer of Sarnen, has proposed to build some bridges such that all the islands are connected. He figured out between which of the islands it is even possible to build a bridge. He already received an offer from Stofl's bridge building company. Mouse Stefanie found out that mouse Benjamin will pay them in Sugus, so she also wants to build a bridge. Furthermore,

- mouse Benjamin will select the bridges such that the total building cost is minimized
- nobody knows what goes on in Benjamin's mind - if there are multiple minimizing options it is not clear which one he will take
- she is only allowed to make an offer for a single bridge

Task

You are given n , the number of islands, m , the number of possible bridges and u_i, v_i, c_i , the start and ending point of the bridges resp the number of Sugus that Stofl offered to build this bridge for. Help mouse Stefanie to find the minimal number of Sugus she can certainly get if she offers to build one bridge.

Example

$n = 3, m = 3, 0, 1, 3; 0, 2, 5; 1, 2, 10$ (Answer: 4; Mouse Stefanie can make an offer for the bridge between node 0 and 2 and get 4 Sugus.)

Fun contest

Mouse Johannes has prepared a fun contest in order to decorate the house. He came up with a rooted tree that consists of n nodes and distributed it to all the participants. He told them to color the leaves of the tree as nicely as possible. When trying to rate the quality of the coloring mouse Johannes had a problem: he realized that he lacks his creative side and does not know which of the colorings looks better, so he came up with the following grading scheme: The participant gets one point for each node where the leaves in the subtree have all pairwise different colors.

Mouse Elias found out about the grading scheme of mouse Johannes, but as he is rather lazy, he does not want to buy too many different colors. So he would like to know how many different colors one needs to get at least k points for $1 \leq k \leq n$. He will then decide how many colors to buy. Unfortunately, he is also too lazy to write a program to determine this answer, so he asks you to help him.

Task

You are given n and for $1 \leq i \leq n - 1$ the parent of node i denoted as p_i (node 0 is the root of the tree). For each $1 \leq k \leq n$ find the number of colors mouse Elias needs to score at least k points with his picture.

Example

$n = 3, p_1 = 0, p_2 = 0$ (Answer: 1 1 2; If we color node 1 and 2 with the same color, Elias earns a point for both nodes 1 and 2. If he wants to score 3 points, he needs to color node 1 and 2 in different colors such that he will also earn the point for node 0)
 $n = 5, p_1 = 0, p_2 = 0, p_3 = 2, p_4 = 2$ (Answer: 1 1 1 2 3)



Increasing

Let $D_{n,k}$ be a list of all *increasing* sequences of n integers whose elements are between 1 and k . For example:

$$D_{3,4} = (1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)$$

$$D_{2,5} = (1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)$$

Within each $D_{n,k}$ the individual sequences have a fixed order: the *lexicographic order*. This order is used in both examples above. Formally, the lexicographic order is defined as follows:

The sequence a_0, \dots, a_{n-1} is *lexicographically smaller* than the sequence b_0, \dots, b_{n-1} if there exists an index $j \geq 0$ such that $a_j < b_j$ and for all $i < j$ we have $a_i = b_i$.

For example, $(3, 7, 12)$ is lexicographically smaller than $(3, 8, 9)$. Hence, in $D_{3,20}$ the sequence $(3, 7, 12)$ would appear sooner than $(3, 8, 9)$. The sequence $(3, 7, 12)$ is also smaller than $(4, 5, 6)$.

Task

You are given n , k , and a sequence S which is one of the sequences in $D_{n,k}$. Your task is to find the index of S in $D_{n,k}$. (The first sequence in $D_{n,k}$ has index 1.)

For example, in $D_{3,2}$ the index of $(1, 2, 4)$ is 2 and the index of $(2, 3, 4)$ is 4.

Example

$n = 5, k = 8, S = [1, 2, 3, 4, 8]$ (Answer: 4; the first four sequences in $D_{5,8}$ are $(1, 2, 3, 4, 5)$, $(1, 2, 3, 4, 6)$, $(1, 2, 3, 4, 7)$, and **$(1, 2, 3, 4, 8)$** .)